1. (a) \[ E[X|\sigma(Y,Z)] = \Sigma_{x,yz} \Sigma_{yz}^{-1} \left( \begin{array}{c} Y \\ Z \end{array} \right) = \frac{(a, 0) \left( \begin{array}{cc} 1 & -b \\ -b & 1 \end{array} \right) \left( \begin{array}{c} Y \\ Z \end{array} \right)}{1 - b^2} = \frac{aY - abZ}{1 - b^2}. \]

(b) \( Y|\sigma(Z) \sim N(bZ, 1 - b^2) \) which implies \( E[Y^2|\sigma(Z)] = b^2Z^2 + 1 - b^2. \)

(c) \[ E[XYZ^2] = E[YZ^2E[X|\sigma(Y,Z)]] \]
\[ = E \left[ \frac{YZ^2(aY - abZ)}{1 - b^2} \right] \]
\[ = \frac{aE[Y^2Z^2] - abE[YZ^3]}{1 - b^2} \]
\[ = \frac{aE[Z^2E[Y^2|Z]] - abE[Z^3E[Y|Z]]}{1 - b^2} \]
\[ = \frac{aE[b^2Z^4 + (1 - b^2)Z^2] - ab^2EZ^4}{1 - b^2} \]
\[ = a. \]

2. Suppose \( \exists A \in \mathcal{A} : \mu(A) = 0 \) but \( \int_A Xd\mu > 0. \) Then \( \exists \) a sequence of simple functions \( 0 \leq X_n \uparrow X \) such that \( \int_A X_n d\mu \to \int_A Zd\mu. \) But this implies, for some \( \delta > 0 \) and integer \( n < \infty, \)
\[ \delta \leq \int_A X_n d\mu = \sum_{j=1}^{k_n} x_{jn}\mu(A \cap B_{jn}) = 0, \]
which is a contradiction! Thus the desired result holds.

3. (a) Showing \( \nu \) is measurable follows from its easy-to-verify additivity on finite disjoint sets combined with the Caratheodory Extension Theorem. Now we will show that it is also \( \sigma \)-finite. Since \( \mu \) is \( \sigma \)-finite, \( \exists \) a countable collection \( A_1, A_2, \ldots \in \mathcal{A} \) such that \( \bigcup_{j=1} A_j = \Omega \) and \( \mu(A_j) < \infty \) for all \( j. \) Now consider the countable collection of sets \( \{ A_j \cap B_k : j, k \geq 1 \}, \) and note that the union of these sets is \( \Omega. \) Moreover, \( \int_{A_j \cap B_k} Yd\mu \leq k\mu(A_j) < \infty. \) Similar arguments verify that \( \lambda \) is also a \( \sigma \)-finite measure.
(b) Suppose $\mu(A) = 0$. Then $\nu(A) = 0$ by the result of Problem 2. Also, if $\nu(A) = 0$, then $\lambda(A) = \int_A Xd\nu = 0$ by reapplication of the Problem 2 result.

(c) Existence follows from $\mu$, $\nu$ and $\lambda$ all being $\sigma$-finite measures (although we only need $\sigma$-finiteness for $\mu$ and $\nu$ at this point but will need it for $\lambda$ later). The forms are $\frac{d\nu}{d\mu} = Y$, $\frac{d\lambda}{d\mu} = XY$, and $\frac{d\lambda}{d\nu} = X$.

(d) i. Suppose $\lambda(A) = 0$. Then

$$\nu(A) = \int_A Yd\mu = \int_{A\cap C} Yd\mu + \int_{A\cap B\cap C} Yd\mu = \int_{A\cap B\cap C} Yd\mu,$$

since $\int_{A\cap B^c\cap C} Yd\mu = 0$ from the fact that $\mu(B^c \cap C) = 0$ and the result of Problem 2. However,

$$\int_{A\cap B\cap C} Yd\mu = \int_{A\cap B} Yd\mu = \int_A [X^{-1}\{X > 0\}] XYd\mu = \int_A Ud\lambda,$$

where $U = X^{-1}\{X > 0\}$ is measurable. Now $\int_A Ud\lambda = 0$ by reapplication of the Problem 2 result. Thus $\nu \ll \lambda$.

ii. Since $\lambda$ is $\sigma$-finite by part 3.(a), we have that $\frac{d\nu}{d\lambda}$ exists and equals $X^{-1}\{X > 0\}$.