BIOS760: 2013 FALL SEMESTER FINAL EXAM

1. Let \( X_1, \ldots, X_n \) be \( n \) i.i.d observations from \( \text{Poisson}(\lambda) \) where \( \lambda \) is an unknown and positive parameter.

   (a) (5 points) Find the UMVUE for \( e^\lambda \).

   (b) (5 points) What is the information bound for \( e^\lambda \)? Does the UMVUE attain this bound?

   (c) (3 points) Derive the MLE for \( e^\lambda \) and its asymptotic distribution.

   (d) (2 points) What is the relative efficiency of the MLE with respect to the UMVUE, which is defined as the ratio between the variance of the UMVUE and the variance of the MLE?

2. We wish to observe \( n \) pairs of observations, \((Y_1, X_1), \ldots, (Y_n, X_n)\), from \( n \) i.i.d subjects. However, some subject’s \( X_i \)’s are missing and we use \( R_i \) to denote non-missingness of \( X_i \) (\( R_i = 1 \) indicates \( X_i \) observable; otherwise, \( R_i = 0 \)). Then then the observed data can be written as

\[(Y_i, R_iX_i, R_i), \quad i = 1, \ldots, n.\]

Assume

\[Y_i = \beta X_i + \epsilon_i, \quad X_i \sim N(0, 1), \quad \epsilon_i \sim N(0, \sigma^2),\]

and \( \epsilon_i \) and \( X_i \) are independent. Furthermore, the conditional probability of \( R_i \) given \((Y_i, X_i)\) may depend on \((Y_i, X_i)\).

   (a) (5 points) Write down the observed likelihood function. You must include the distribution of \( R \) given \((Y, X)\) in the likelihood function.

   (b) (5 points) Assume that \( R_i \) only depends on \( Y_i \). We treat the unobserved \( X \)’s, i.e., those subjects with \( R_i = 0 \), as missing data. Write down the EM algorithm for estimating parameters \((\beta, \sigma^2)\).

   (c) (5 points) If \( R_i \) depends on \( X_i \) and particularly, \( R_i = I(X_i > 0) \), what is the EM algorithm for computing the maximum likelihood estimator for \((\beta, \sigma^2)\)? You can leave the expressions in the algorithm.