1. (a) Using the change of variable 
\[ v = x - \theta, \]
we obtain that
\[ \int_R f_\theta(x)dx = \int_0^\infty e^{-(x-\theta)}dx = \int_0^\infty e^{-v}dv = 1. \]
The full likelihood has the form
\[ \prod_{i=1}^n f_\theta(X_i) = \exp \left[ \sum_{i=1}^n X_i \right] e^{n\theta}1\{X_1 > \theta\} = a(X_1, \ldots, X_n)b(\theta, X_1), \]
and thus \( X_1 \) is sufficient by factorization.

(b) The density of the smallest order statistic in an i.i.d. sample with density \( f_\theta(x) \), where we let \( F_\theta(x) \) be the corresponding c.d.f., is
\[ nf_\theta(x)(1 - F_\theta(x))^{n-1} = ne^{-(x-\theta)}\exp[-(n-1)(x - \theta)], \]
and the given form of the density follows. Now let \( \gamma(x) \) be an arbitrary measurable integrable function, and note that for any \( \theta \in R \),
\[ \int_R \gamma(x)g_\theta(x)dx = 0 \Rightarrow \int_\theta^\infty \gamma(x)ne^{-n(x-\theta)}dx = 0, \]
which implies that
\[ \int_\theta^\infty \gamma(x)e^{-nx}dx = 0 \Rightarrow \gamma(\theta)e^{-n\theta} = 0, \]
where the last equality follows from differentiating both sides of the left-hand equality with respect to \( \theta \). This now implies that \( \gamma(\theta) = 0 \) for all \( \theta \in R \). Thus \( X_1 \) is complete since \( \gamma(x) \) was arbitrary.

(c) The expectation of \( X_1 \) is
\[ \int_\theta^\infty xne^{-n(x-\theta)}dx = \theta + \int_R n(x - \theta)e^{-n(x-\theta)}dx \]
\[ = \theta + n^{-1}\int_0^\infty ve^{-v}dv \]
\[ = \theta + n^{-1}, \]
where the second-to-last equality uses the change of variables \( v = n(x - \theta) \). Thus, since \( X_1 \) is complete and sufficient for \( \theta \), the UMVU of \( \theta \) is \( X_1 - n^{-1} \).
2. (a) Since the log-likelihood for a single observation is $\ell(p) = \log[pU_1 + (1 - p)(1 - U_1)]$, the corresponding score and information for the sample are

$$\hat{\ell}_n(p) = \sum_{i=1}^{n} \frac{2U_i - 1}{pU_i + (1 - p)(1 - U_i)}$$

and

$$I_n(p) = -\hat{\ell}_n(p) = \sum_{i=1}^{n} \frac{(2U_i - 1)^2}{[pU_i + (1 - p)(1 - U_i)]^2}.$$

(b) The result follows directly from the standard Newton-Raphson iteration formula for the sample log-likelihood:

$$p^{(k+1)} = p^{(k)} + \left[I_n(p^{(k)})\right]^{-1} \hat{\ell}_n(p^{(k)}).$$

(c) The form of $m_p(\delta, u)$ follows directly from the mixture structure. The marginal density for $U$ is obtained by summing over $\delta$ (corresponding to the $\Delta$ component) which yields that the marginal is $h_p(u)$. The form of the sample log-likelihood follows directly from the log-likelihood for a single observation obtained from $m_p(\delta, u)$, which is

$$\log(2) + \Delta [\log(p) + \log(U)] + (1 - \Delta) [\log(1 - p) + \log(1 - U)].$$

(d) The expectation of $\Delta_i$, given $U_i$ at the value $p = p^{(k)}$, is the same as the probability that $\Delta_i = 1$, given $U_i$ evaluated at $p = p^{(k)}$, which is

$$\frac{m_p^{(k)}(1, U_i)}{\sum_{\delta=0,1} m_p^{(k)}(\delta, U_i)} = \frac{p^{(k)}U_i}{p^{(k)}U_i + (1 - p^{(k)})(1 - U_i)}.$$

(e) The expectation of the full sample log-likelihood, with individual components having the from in (c) above, given the observed data, after throwing out the constant $\log(2)$, has the form

$$E \left[ \sum_{i=1}^{n} \Delta_i [\log(p) + \log(U_i)] + (1 - \Delta_i) [\log(1 - p) + \log(1 - U_i)] \right| Y_{obs}, p^{(k)}]$$

$$= \sum_{i=1}^{n} E \{ \Delta_i [\log(p) + \log(U_i)] + (1 - \Delta_i) [\log(1 - p) + \log(1 - U_i)] | U_i, p^{(k)}\}$$

$$= \sum_{i=1}^{n} W_k(U_i) [\log(p) + \log(U_i)] + (1 - W_k(U_i)) [\log(1 - p) + \log(1 - U_i)],$$
where the second equality follows from the independence across observation. Now differentiate the last term above with respect to $p$, to obtain that the next EM update $p^{(k+1)}$ is the solution of

$$\sum_{i=1}^{n} \left[ \frac{W_k(U_i)}{p} - \frac{1 - W_k(U_i)}{1 - p} \right] = 0,$$

and the desired conclusion follows.

(f) In the EM algorithm, if $p^{(k)} = 0$, then $p^{(k+1)} = 0$ and the EM iteration is stuck at zero. For the Newton-Raphson iteration, $p^{(k+1)}$ is almost surely not zero if $p^{(k)} = 0$. This says that the EM has faulty local solutions which need to be avoided or worked around in the estimation process. The Newton-Raphson algorithm appears to not have this problem.