The joint density is given as

\[(2\theta)^{-n}I(|X_1| \leq \theta, \ldots, |X_n| \leq \theta) = (2\theta)^{-n}I(|X|_{(n)} \leq \theta)\].

Thus, \(|X|_{(n)} \) is sufficient. Suppose \(E[|Q(|X|_{(n)})\)] = 0\). Since

\[P(|X| \leq x) = x^n/\theta^nI(0 \leq x \leq \theta) + I(x \geq \theta),\]

we have

\[E[Q(|X|_{(n)})] = \int_{\theta}^{\infty} Q(x)x^{n-1}/\theta^n dx = 0.\]

That is, \(\int_{\theta}^{\infty} Q(x)x^{(n-1)}dx = 0\) so after taking derivatives, we obtain \(Q(\theta) = 0\). This implies that \(|X|_{(n)} \) is also complete.

(b) Note that if \(x \leq 0\), \(P(n(1 - |X|_{(n)}/\theta) \leq x) = 0\); if \(x > 0\), for \(n \) large enough,

\[P(n(1 - |X|_{(n)}/\theta) \leq x) = P(|X|_{(n)} \geq \theta - \theta x/n) = 1 - (1 - x/n)^n \rightarrow 1 - e^{-x}.\]

We conclude that \(n(1 - |X|_{(n)}/\theta) \) converges in distribution to the exponential distribution.

(c) Clearly, the MLE for \(\theta \) is \(|X|_{(n)}\). Thus, the MLE for \(g(\theta) \) is \(g(|X|_{(n)})\). By the delta method, we obtain

\[n(g(\theta) - g(|X|_{(n)}))/\theta \rightarrow_d g'(\theta)\text{Exp}(1).\]

Thus, \(n(g(\theta) - g(|X|_{(n)})) \rightarrow_d \theta g'(\theta)\text{Exp}(1)\). That is, the limiting distribution is the exponential distribution with mean \(\theta g'(\theta)\). Let \(t_{1-\alpha} \) be the \((1 - \alpha)\)th quantile of \(\text{Exp}(1)\), i.e., \(t_{1-\alpha} = -\log(\alpha)\). Then by estimating \(\theta \) using \(|X|_{(n)}\), a confidence interval for \(g(\theta) \) is \([0, g(|X|_{(n)}) + |X|_{(n)}g'(|X|_{(n)})t_{1-\alpha}/n]\).

(d) To find the UMVUE, we need to determine \(Q(|X|_{(n)})\) satisfying \(E[Q(|X|_{(n)})] = g(\theta)\). Equivalently,

\[\int_{\theta}^{\infty} Q(x)x^{n-1}dx = g(\theta)\theta^n.\]

Taking derivatives on both sides, we obtain

\[Q(x) = g(x) + g'(x)x/n.\]

The UMVUE for \(\theta \) is \(T_n = g(|X|_{(n)}) + g'(|X|_{(n)})|X|_{(n)}/n\).

(e) We note

\[n(g(\theta) - T_n) = n(g(\theta) - g(|X|_{(n)})) - g'(|X|_{(n)})|X|_{(n)}.\]

Since the previous result implies that \(|X|_{(n)} \rightarrow_p \theta\), we obtain from the Slutsky lemma that

\[n(g(\theta) - T_n) \rightarrow_d \theta g'(\theta)\text{Exp}(1) - g'(\theta)\theta.\]

A confidence interval for \(g(\theta) \) is \([0, T_n + |X|_{(n)}g'(|X|_{(n)})t_{1-\alpha}/n - g'(|X|_{(n)})|X|_{(n)}]\).
2. (a) The conditional density is given as
\[ \int_\xi e^{\beta X \xi} \exp\{-Ye^{\beta X / \lambda}\} \Gamma(\lambda)^{-1} \lambda^{\lambda-1} \exp\{-\lambda \xi\} d\xi = e^{\beta X (1 + Ye^{\beta X / \lambda})^{-1}}. \]

Thus, the likelihood function is given as
\[ \prod_{i=1}^{n} \left[ e^{\beta X_i (1 + Y_i e^{\beta X_i / \lambda})^{-1}} f(X_i) \right], \]
where \( f(x) \) is the density of \( X \).

(b) Assume that \( X \) has a full rank with positive probability. Then \( \beta \) is identifiable since
\[ e^{\beta X_i (1 + Y_i e^{\beta X_i / \lambda})^{-1}} f(X_i) = \left[ e^{\beta X_i (1 + Y_i e^{\beta X_i / \lambda})^{-1}} f^*(X_i) \right] \]
leads to \( \beta X_i = \beta^* X_i \).

(c) It follows from
\[ E[Y|X] = E[Y|X, \xi]|X] = E[e^{\beta X \xi - 1}|X] = e^{\beta X c}, \]
where \( c = E[\xi^{-1}] = \lambda/(\lambda - 1) \). From the results for estimating equation, if \( \hat{\beta} \) is the solution, then \( \sqrt{\hat{\beta} - \beta} \) converges in distribution to a normal distribution with mean zero and variance (sandwich formula)
\[ E[cX^2 e^{-\beta X}]^{-1} E[X^2(Y - ce^{-\beta X})^2] E[cX^2 e^{-\beta X}]^{-1}. \]

(d) The score equation is
\[ \sum_{i=1}^{n} \left[ X_i - \frac{(\lambda + 1)Y_i X_i e^{\beta X_i}}{\lambda + Y_i e^{\beta X_i}} \right] = 0. \]

The Newton-Raphson iteration is
\[ \beta^{(k+1)} = \beta^{(k)} - \left\{ -\sum_{i=1}^{n} \frac{(\lambda + 1)Y_i X_i^2 e^{\beta^{(k)} X_i}}{\lambda + Y_i e^{\beta^{(k)} X_i}} \right\}^{-1} \left\{ \sum_{i=1}^{n} \frac{(\lambda + 1)Y_i^2 X_i^2 e^{2\beta^{(k)} X_i}}{(\lambda + Y_i e^{\beta^{(k)} X_i})^2} \right\} \]
\[ \times \sum_{i=1}^{n} \left[ X_i - \frac{(\lambda + 1)Y_i X_i e^{\beta^{(k)} X_i}}{\lambda + Y_i e^{\beta^{(k)} X_i}} \right] = 0. \]

(e) The complete log-likelihood function (concerning \( \beta \)) is
\[ \prod_{i=1}^{n} \left[ e^{\beta X_i \xi_i} \exp\{-Y_i e^{\beta X_i \xi_i}\} \right]. \]

Then the M-step solves equation
\[ \sum_{i=1}^{n} \left[ X_i - Y_i e^{\beta X_i} X_i w_i \right] = 0. \]

In the E-step, we calculate \( w_i \) as the conditional expectation of \( \xi_i \) given observed data: since \( \xi|Y_i, X_i \) follows a gamma distribution with parameter \((1 + \lambda, [Y_i e^{\beta X_i} + \lambda]^{-1})\),
\[ w_i = E[\xi|Y_i, X_i] = (1 + \lambda)/[Y_i e^{\beta X_i} + \lambda]. \]