Gene expression

A powerful and flexible approach to the analysis of RNA sequence count data

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1 INTRODUCTION

Sequencing of RNA-based libraries (‘RNA-Seq’) can provide digital gene expression measurement, and is an attractive approach, potentially replacing microarrays for analyzing the transcriptome in an unbiased and comprehensive manner. For genes with very low or very high levels, microarrays often lack sensitivity, or can result in saturated signal. In contrast, RNA-Seq has been shown to have high accuracy across many orders of expression magnitude (Marioni et al., 2008), with clear advantages over microarray hybridization. At a basic level, simple counts of RNA sequences can be used for digital gene expression measurement, and are the subject of this article. Additional information derived from the sequences, such as de novo exon discovery, are beyond our scope, although many of the considerations here may be applied to the deeper study of sequence content.

RNA-Seq technology is currently more expensive than comparable array technologies, and thus the sample sizes are typically small. In addition, even as the cost of RNA-Seq profiling drops, the precision of the technology will enable sensitive investigation of small samples (for example, pairwise comparisons among many experimental conditions examined). Eventually, however, we expect that sequence-based transcriptional profiling will become the standard, with large datasets becoming affordable. Thus, there is a pressing need for sensitive statistical approaches that can accommodate large variation in the available sample sizes. RNA-Seq count data consists of the number of instances that each transcript has been sequenced, arising from random sampling events for a large number of sequences (the library size). The simplest data model may be multinomial, with probability proportional to the true expression level. These probabilities are small and counts are accumulated over many reads, so Poisson approximations are commonly used. However, it has been repeatedly shown that RNA-Seq data are overdispersed (Robinson et al., 2010)—i.e. the variance of sequence counts tends to be greater than would be expected for multinomial or Poisson data. Thus, any careful analysis of the data, and in particular any differential expression analysis, must account for this overdispersion. Additional factors, such as the length of the transcript and potential sequencing bias, are important in performing inference on absolute expression levels, but here we are concerned primarily with comparison of expression values across different samples. Before proceeding to our testing framework, we briefly review the available methods for performing differential expression analysis for RNA-Seq data.

The package edgeR (Robinson et al., 2010) was initially designed as a penalized approach to identify differences between two sample groups. The current version has a variety of penalized overdispersion approaches, including ‘common’ penalized dispersion, a ‘tagwise’ approach that shrinks individual genes/tags and the tagwise procedure with a trend as a function of expression level. A negative binomial model is used, which essentially corresponds to an overdispersed Poisson model. The approach uses empirical Bayes methods to moderate the degree of overdispersion, with the aim of reducing error in a similar manner as penalized methods in microarray analysis (Smyth, 2004). The baySeq approach (Hardcastle and Kelly, 2010) is more explicitly Bayesian, also assuming negative binomially distributed count data, and can use the data to elicit a prior for the overdispersion parameters. BaySeq...
Analysis of future, more complex datasets will require more flexible software for handling RNA-Seq data, and our power/FDR results indicate that straightforward beta-binomial modeling has favorable characteristics. In contrast to the competing penalization approaches, we find only modest advantages for penalization, which is mainly useful for very small sample sizes.

2 METHODS

2.1 Mean–overdispersion modeling

The 60 HapMap CEU RNA-Seq samples from Montgomery et al. (2010) (~20000 genes, described in detail below) are used to illustrate the overdispersion typical of such datasets. Figure 1 shows the sample variance versus the sample mean on the log–log scale for the read counts across the samples. The relationship between the mean and variance is very strong, and the overdispersion increases with mean expression, as evidenced by the increasing gap between the data points and the unit line which corresponds to a Poisson assumption. The pattern remains essentially unchanged if the counts are standardized by the library size per sample (data not shown).

A similar plot using a random subset of five samples shows the same pattern (Supplementary Fig. S1). The data illustrate that overdispersion is an important feature of the data, and can either be fit as a separate parameter or in a model for the dependence of the overdispersion on the average expression. Discussions of RNA-Seq data often focus on the ‘length bias’, the phenomenon that longer transcripts are more likely to contain mapped reads. For example, Oshlack and Wakefield (2009) point out differing mean–variance relationships for shorter versus longer genes. Supplementary Figure S2 illustrates that, at least for datasets analyzed here, the mean is a stronger determinant of the variance (and overdispersion) than the length (also see Methods and Results in Supplementary Material). In addition, we are mainly interested in comparing expression levels within genes, across conditions) can produce missing values or spurious tests. The vast majority of publications have used purely simulated data, or small example datasets for which comparative conclusions are difficult.

In this article, we describe DESeq, a comprehensive approach to the analysis of RNA-Seq transcriptional count data. DESeq employs a beta-binomial model for the count data, corresponding to the view that the observation of a sequence for a particular transcript is a beta-binomial model for the count data, corresponding to the view that the observation of a sequence for a particular transcript is a Bernoulli random variable with an intrinsic probability for each sample. These probabilities are allowed to vary according to a beta distribution, thus allowing for overdispersion, with a mean that depends on the design variables/covariates. As the library size is large, the beta-binomial behaves similarly to an overdispersed Poisson. We thus expect that the beta-binomial provides similar fits for the variance compared to the binomial, after considering design effects. We will use \( \theta \) to represent the library size for the \( j \)-th sample. Marginally, the likelihood is

\[
\logit(\frac{E(y_{ij})}{1-E(y_{ij})}) = \beta_0 + \beta_1 x_{ij} + \ldots + \beta_p x_{pj} = X_{ij} \beta
\]

(1)

for the \( p \times 1 \) matrix of regression coefficients \( X = [x_{i0}, \ldots, x_{ip}] \). \( \theta_j \) follows a Beta distribution, parameterized so that its variance is \( \phi \sqrt{E(y_{ij})(1-E(y_{ij}))} \). Values \( \phi > 0 \) correspond to overdispersion compared to the binomial, after considering design effects. We will use \( \phi_j = \sum_{i} y_{ij} \) to represent the library size for the \( j \)-th sample. Marginally, the likelihood is

\[
f(y_{ij} | \beta, \alpha) = \binom{\theta_j}{y_{ij}} \theta_j^{y_{ij}} (1-\theta_j)^{\alpha_j-y_{ij}}
\]

(2)

where \( \theta_j \) is the Beta function, \( \alpha_j = \theta_j (1-\phi_j) / \phi_j \), and \( \alpha_j = (1-\phi_j) / \phi_j \).

Parameter estimation is performed by maximum likelihood, using either of two approaches:

(i) the free model, in which \( \phi_j \) is estimated as a ‘free’ parameter separately for each gene; and

(ii) the constrained model, in which \( \phi_j \) is estimated using an assumed mean–overdispersion relationship. For the constrained model, it might
be reasonable for the overdispersion to also depend on the sample \( j \), but for simplicity and comparability with the free model it is simpler to assume a single \( \phi \) for each gene \( i \). Each \( \phi_i \in [0,1] \), and so it is convenient to work with a logistic transformed parameter, and we adopt the polynomial relationship

\[
\hat{\gamma} = \hat{\phi} = \sum_{k=0}^{K} [\text{mean}(X)|B_i] \hat{\phi}_k.
\]

where the mean is over the \( n \)-vector \( XBi \). Note that the logit specification does not allow for underdispersion, which would be biologically implausible. In practice, a low degree polynomial, with \( K \geq 3 \), appears to provide an adequate fit, and we use simple plug-in estimates \( \hat{B}_i \) and \( \hat{\phi}_k \) from the free model to obtain least squares estimates of the \( \gamma \) values. The \( \gamma \) values are assumed to be the same across the genes. This approach is similar to and generalizes a mean–variance modeling approach for expression microarrays (Hu and Wright, 2007), which had previously been performed only for two-sample experiments. The intent of the modeling is to increase power to detect differential expression for small sample sizes. Note that the estimation of the mean–overdispersion relationship does not reduce the degrees of freedom for individual genes, as all genes are used to estimate the few \( \gamma \) parameters.

Supplementary Figure S3A shows the \( \hat{\gamma} \) values obtained from the free model for 6 versus 6 samples from the CEU dataset, with equal number of males and females in each group, plotted against mean(\( X \)). The relationship is reasonably polynomial, inspiring the constrained model, with much of the variation in \( \hat{\gamma} \) explained (multiple \( R^2 = 0.60 \) for a cubic fit). Moreover, the variation in the \( \hat{\gamma} \) residuals can mostly be attributed to sampling variation consistent with the \( \hat{\gamma} \) standard errors (Supplementary Fig. S3B).

Optimization for both models is performed using the R optm function, with starting values for \( B \) obtained from linear regression and for \( \hat{\gamma} \) from marginal evidence of overdispersion (without considering design effects). Finally, we note that real data can produce outlying estimates for a small percentage of genes, especially for the constrained model and large sample sizes. Thus, we have devised very simple outlier detection/correction approaches to avoid spurious results (Methods and Results in Supplementary Material).

### 2.3 Testing and design matrices

The free and constrained models both provide considerable flexibility, as the design matrix \( X \) is arbitrary and can be specified by the user. The statistical significance of any covariate can then be determined from the corresponding Wald statistic (the estimate of differential expression divided by its standard error). The vast majority of published RNA-Seq studies involve simple two-sample comparisons, so the primary testing is on \( \hat{\beta}_j \) in each \( B_j \), corresponding to the indicator column in \( X \) representing group assignment. Empirical investigation in small samples indicates that the Wald statistic \( \hat{\beta}_j / \text{SE}(|\hat{\beta}_j|) \) for the free model under the null hypothesis (and standard normal for constrained), with adjustments for zero counts in the data (Methods and Results in Supplementary Material), and we employ this approximation for two-sample testing.

Many future studies are likely to focus on a few (perhaps multilevel) factors, as is performed in ANOVA analysis. Thus, BBSeq has been designed with a simple function to generate design matrices corresponding to multilevel factors. Moreover, BBSeq performs likelihood ratio comparisons for the overall statistical significance of each included factor. A more detailed description is provided in Methods and Results of Supplementary Material for the CEU HapMap samples, along with other data downloaded in March 2010.

### 2.4 HapMap RNA-Seq data sources

The CEU HapMap lymphoblastoid RNA-seq dataset of Montgomery et al. (2010) was obtained from their website (60 samples, http://jungle.uniqx.ch/ncaseq/CEU00/) as mapped tags (these and all other data downloaded in March 2010). ReSeq annotation for 21,498 genes (corresponding to 32,644 transcriptional isoforms) from the hg18 genome build was downloaded from the UCSC Genome Browser (http://genome.ucsc.edu/), with gene/exon boundaries used for a transcript database. Mismatches of up to two bases were allowed. Counts were obtained by summing RNA-Seq reads mapping to the exons of each ReSeq entry, and ReSeq IDs further annotated to the official gene symbol. An average of 9.8 million reads per sample was mapped. Mapping of reads to multiple transcript isoforms was kept in the dataset for completeness. Differential expression analyses using only the most-common isoforms for each gene versus retaining all isoforms as if they were separate genes resulted in nearly identical inference. Mapped reads from the Argonne HapMap YRI dataset (69 unique samples), Pickrell et al. (2010) were downloaded from http://apl1.uuchicago.edu/RNA_Dseq_data/ mapped_reads/, and applied to hg18 using the same procedures as performed for the CEU data (an average of 4.3 million mapped per sample). The total number of genes containing mapped reads in the CEU dataset was 20,904 (32,027 with redundant isoforms included), and in the YRI dataset was 20,488 (31,508).

### 2.5 Simulated data and subsampling from real datasets

For simulated datasets, as well as analysis of the HapMap data, the two BBSeq models were compared with other approaches, including the three edgeR models, DESeq, baySeq and a quasi-likelihood overdispersed binomial GLM (detailed descriptions of Methods in Supplementary Material).

**Dataset 1:** The first dataset consists of 100 independent simulations of 10,000 genes with 5 versus 5 samples for two-sample comparisons, reported in Hardcastle and Kelly (2010) (under ‘Random dispersion simulations’). A known 10% of the genes were differentially expressed with a ratio of average count levels of \((e_{i1})/e_{i2}\) between two experimental groups. The data were obtained from the authors, who used edgeR parameter estimates from...
a SAGE dataset (Zhang et al., 1997). However, it is not clear whether the simulation setup mimics current RNA-Seq data.

Dataset 2: the second dataset consisted of our own simulations of two-sample comparisons of groups 1 and 2 ($n_1=5$ versus $n_2=5$ or $n_1=2$ versus $n_2=2$), with average expression levels matched to that of a real RNA-Seq dataset. For a two-sample experiment, the coefficient matrix is $B=(b_0, b_1)^T$ (suppressing the subscript $i$). We used the parameterization

$$r = \frac{\beta_0 + \beta_1}{\beta_0 - \beta_1}$$

(4) to control the degree of differential expression, which is interpretable as the odds ratio for the expected read probabilities in group 2 versus group 1. Values $r > 1$, $r < 1$ and $r = 1$ correspond to group 2 having greater, lower and equal average expression, respectively, as group 1. To obtain empirically driven parameter values, we first drew random subsamples of the CEU data (Montgomery, 2010), and for each subsample ran the free and constrained models to obtain $\beta_0$ and $\gamma$ estimates for each gene. Then for each value $r$, 20 simulations were performed following Equation (3), treating the estimated values as true parameters, with 10% of the genes chosen to be differentially expressed (i.e. with $r$ at the alternate value), which together with $\beta_0$ determined the corresponding $\beta_1$. Then the data were simulated according to the corresponding beta-binomial distribution, with library sizes (total number of reads) obtained from the actual samples. Any simulated genes consisting entirely of zero counts across the samples were removed. For the receiver operator characteristic (ROC) curve comparisons, we attempted to make the results as realistic as possible by using, for null genes and each simulation, the actual read counts drawn from a random set we attempted to make the results as realistic as possible by using, for null genes.

Dataset 3: it is difficult for simulations to capture the complexities of real data, but it is also difficult to obtain RNA-Seq datasets for which the truth of differential expression is known. Moreover, results from any single small dataset can be highly variable, and possibly not reflect the overall behavior of a procedure. We reasoned that comparisons of differentially expressed genes between males and females in the HapMap RNA-Seq datasets would be an ideal testing ground for the ability to detect differential expression, and we used subsamples of the CEU and YRI HapMap RNA-Seq data to form our third dataset. Genes on the sex chromosomes would be expected to predominate among those most differentially expressed. Despite chromosomal inactivation, a sizeable number of X-chromosome genes are differentially expressed, with modern estimates of this proportion ranging from ~15% (Carrel and Willard, 2005) to 5% (Johnston et al., 2008). Y chromosome genes should be expressed only in males, but the expression in transformed lymphocytes for many genes may be low. Nonetheless, using the genes on the autosomes as a control, the ability to efficiently detect differential expression on the sex chromosomes can be used to compare procedures, with the degree of differential expression varying widely across this set.

3 SIMULATIONS AND EXAMPLES

3.1 Comparisons with other approaches

3.1.1 Comparisons using Dataset 1 The original authors (Hardcastle and Kelly, 2010) used ROC curves to compare baySeq to a number of other methods, including edgeR. Focusing on the most significant genes and expressed as a false discovery rate produced the result in Supplementary Figure S5, which is directly comparable to the lower right panel of Figure 2 in Hardcastle and Kelly (2010). BaySeq has the best performance, while our two approaches perform similarly to edgeR and DESeq for the most significant genes, but perform more poorly for larger numbers of rejected genes. A careful comparison shows that the free model is similar to the unpenalized

\[ \text{log-linear model in the original baySeq figure, as expected, as is the overdistributed GLM.} \]

The results are perhaps to be expected, as the data follow the idealized simulation conditions for baySeq. Supplementary Figure S5 shows the relationship between log(variance) and log(mean) for the first group in the first simulation. Although there is an apparent mean–overdispersion relationship, note that the dispersion in sample variance is more extreme, especially for genes with low expression than encountered in the CEU data (Fig. 1 and Supplementary Fig. S1). Moreover, the average expression level in the real RNA-Seq datasets tends to be higher than for Dataset 1. This difference is strikingly illustrated in the number of zero counts. For either the CEU or YRI datasets, ~60% of the genes in a subsample of size 10 will show no zero counts across the samples, while for Dataset 1 the value is 17%. It is unclear how these differences affect the performance of these methods with current RNA-Seq data.

3.1.2 Comparisons using Dataset 2 As described earlier, Dataset 2 consists of simulations with $n_1=n_2=2$ and $n_1=n_2=5$, under the model using intercept $\beta_0$ and mean–overdispersion relationships obtained from the CEU data, with parameter $r$ controlling the degree of true differential expression. In each simulation, a random 10% of the genes were used as ‘alternative’. We were interested in power to declare differential expression at $\alpha=0.05$ and the more stringent $\alpha=0.001$. Such a comparison requires interpretable $P$-values, and so we do not show results for baySeq, which provides only posterior probabilities for differential expression. The remaining approaches exhibited reasonable control of type I error, but to make precise power comparisons we also determined the empirical threshold for each approach such that $P_{1\alpha}(\theta < \text{threshold})=\alpha$. An illustrative power curve for $n_1=n_2=2$ and $\alpha=0.05$ is shown in Figure 2. The additional scenarios are shown in Supplementary Figure S6. For these simulations, the constrained model performs best, as might be expected, as the approach is able to accurately estimate overdispersion using the model. The relative improvement in power for the constrained model over the other models is greatest for $n_1=n_2=2$ and $\alpha=0.001$, and for modest effect sizes $r$. For $n_1=n_2=5$, the relative improvement of the constrained model over other penalized approaches is reduced.

For these simulations, the empirical type I error for the nominal $P$-values is shown in Supplementary Table S1. The BBSq models
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show near-nominal type I error, while the other methods do not generally perform as well. Focusing on \( n_1 = n_2 = 5 \) and a moderate effect size \( r = 2.0 \) and using the resampled data counts to create ‘null’ genes as described above, we show ROC curves for the various methods in Figure 3. Examination of ROC curves reveal differing behavior for genes with low expression versus high expression (using the median \( \beta_0 \) estimate as a splitting criterion). Here, the free model outperforms the other models, except for high expression genes, where it is similar to baySeq and to the overdispersed GLM. However, using all genes, the free model appears to be best. Note that these results differ somewhat from the ‘pure’ simulations for the power curves, because the sampling of null genes by random draws from the CEU induces correlations and dispersion behavior that may not be reflect in pure null simulation.

Based on these simulations and potential sensitivity to the vagaries of real data, we propose that the constrained model has value mainly for very small sample sizes (such as \( n_1 = n_2 = 2 \)), with the theoretical advantages for larger sample sizes outweighed by potential model deviations. Thus, we recommend the constrained model only for very small sample sizes.

After consideration of the simulations, we were motivated to compare the free model to competing models for moderate sample sizes and real data. These comparisons are performed in Dataset 3 below.

3.1.3 Comparisons using Dataset 3 Using the sex information for the CEU HapMap and YRI samples, we applied the free model and the competing approaches to 10 random subsamples of \( n_1 = 6 \) males versus \( n_2 = 6 \) females for CEU and YRI separately. For the edgeR approach, we used only the trend penalization procedure, as this had performed generally the best in the power comparisons. For each subsample, we counted the number of sex chromosome genes among the top detected/rejected genes. The average across the 10 subsamples is shown in Figure 4. Here, the free model is best for the CEU data, and is similar to edgeR, DESeq and the GLM for the YRI data. For YRI, the baySeq approach is the most sensitive in detection.

These comparisons are among the most extensive such examinations performed with real data and for which we are able to explore the ‘truth’ of differential expression in the dataset. We emphasize that true differential expression between males and females may occur for some genes on the autosomes. The rationale of our analysis, following current understanding of sex chromosome expression, is merely that genes on the sex chromosomes should be over-represented if a detection procedure is sensitive.

As described in Section 2, for completeness of mapped reads, our analyses were performed using multiple common transcript isoforms (e.g. splicing variants) as if they were separate genes. Many reads map to several isoforms of a gene, technically violating independence assumptions for read counts. RNA-Seq analysis packages typically provide little guidance on this issue. However, the library sizes are typically so large that the inference for any one isoform is essentially the same whether or not the analysis is restricted to unique genes, as illustrated for a 12-sample CEU analysis, shown in Supplementary Figure S7.

3.1.4 Sex-specific expression and outlier sensitivity We also used the five methods to perform differential expression analysis for males versus females for the entire set of 60 CEU samples.
e.g. the top-ranked genes by the free model are also significant by highest among all genes) may make it vulnerable to spurious baySeq females. We speculate that the high expression of the gene (fourth 47 according to DESeq This result is largely driven by a single approach, we examined the most significant autosomal genes, and in the reasons for such strikingly different gene lists. For each genes from the X chromosome among the top 10. We were interested which these methods may be more sensitive. Bayseq identifies three variances within each of males and females (data not shown), to XIST 

\[ n \] does not appear differentially expressed (Supplementary Figure S8), 

\[ P = 1.78E-07 \] has not been widely described, and a literature search did not reveal compelling evidence for sex-specific expression. Nonetheless, the histogram shows a clear trend of higher expression in males. SULF1 

\[ P = 7.82E-42 \] according to edgeR Trend, and \[ P = 8.14E-07 \] according to DESeq This result is largely driven by a single outlying high value in females. BaySeq identifies ACTG1 as the most differentially expressed of all genes, although the count distributions overlap almost entirely, with the two highest values occurring in females. We speculate that the high expression of the gene (fourth highest among all genes) may make it vulnerable to spurious baySeq findings, but further investigation is warranted. Beyond the potential sensitivity to outliers, most of the methods are in broad agreement—e.g. the top-ranked genes by the free model are also significant by the other methods, but appear further down on their respective lists. The entire list of genes and \( P \) values for all methods are provided on the software website.

4 CONCLUSION AND DISCUSSION

We have described a procedure to implement beta-binomial modeling of RNA-Seq tag counts. For the free-\( \phi \) model, our procedure is somewhat similar to other overdispersed generalized linear models. However, in our simulations and in the HapMap data, the direct parametric modeling of the overdispersion parameter appeared to be advantageous. Moreover, our BBSeq software simplifies the analysis for researchers less familiar with modeling and construction of design matrices, and issues such as outlier detection are handled automatically. The constrained-\( \phi \) approach, while still very simple, has potential advantages in the analysis of very small datasets, which remain very common. We emphasize that both procedures offer much more flexible handling of design variables and other covariates than competing purpose-built procedures.

A surprising result from our investigation is that it is unclear for modest size samples (say 5 or more per sample group) that the careful attention to penalization procedures, which are implicit in both the competing procedures and in our constrained approach, are truly necessary for effective inference. A better understanding of the true nature of differential expression may be necessary in order to fully understand these issues. Much of the motivation behind penalized approaches lies in a notion that genes with low expression have an unfavorable ratio of signal to noise. As the accuracy of expression profiling further improves, this notion may be replaced by a deeper understanding of the degree of differential expression need to produce biologically important changes, which may depend on baseline expression level as well as other contextual information. Our analysis of male/female differential expression in the entire CEU dataset was intended only as a simple illustration, but highlights a possible sensitivity to outliers of shrinkage/penalization methods, and deserves further investigation.

Several investigators have pointed out that a relatively small number of genes can be responsible for large variations in total read counts (Robinson and Oshlack, 2010; Bullard et al., 2008), and thus it is conceivable that a gene at constant mRNA concentration need to produce biologically important changes, which be replaced by a deeper understanding of the degree of differential expression of various isoforms (Blekhman et al., 2010). Sequence reads that are otherwise uninformative about allele specificity or varying isoforms still provide evidence of overall expression level, which in turn may indirectly inform a deeper understanding of expression changes.

Fig. 4. Number of detected genes on the sex chromosomes versus number of genes detected, using subsamples from Dataset 3 with \( n_1 = 6 \) males versus \( n_2 = 6 \) females. (A) The CEU dataset. (B) the YRI data.


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### Note Added in Proof: The advance access version of this manuscript included an incorrect plot of the baySeq results in Figure 4. We regret the error. In addition, analysis of the full CEU dataset by an updated version of baySeq (version 1.6) is reported to identify fewer outliers (Dr. Thomas Hardcastle, personal communication).

### REFERENCES


