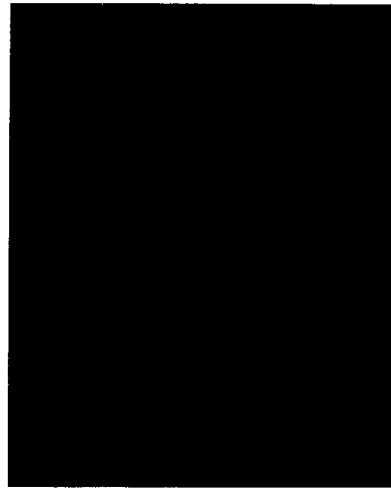




Statistical Inference

Second Edition

George Casella
Roger L. Berger

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