# Appendix C: Statistical Tables

Table C.1 *Cumulative distribution function of the standard normal distribution*
(*a*) *For x in* 0.1 *intervals*

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|
| 0.0 | 0.5000 | 1.3 | 0.9032 | 2.6 | 0.9953 |
| 0.1 | 0.5398 | 1.4 | 0.9192 | 2.7 | 0.9965 |
| 0.2 | 0.5793 | 1.5 | 0.9332 | 2.8 | 0.9974 |
| 0.3 | 0.6179 | 1.6 | 0.9452 | 2.9 | 0.9981 |
| 0.4 | 0.6554 | 1.7 | 0.9554 | 3.0 | 0.9987 |
| 0.5 | 0.6915 | 1.8 | 0.9641 | 3.1 | 0.9990 |
| 0.6 | 0.7257 | 1.9 | 0.9713 | 3.2 | 0.9993 |
| 0.7 | 0.7580 | 2.0 | 0.9772 | 3.3 | 0.9995 |
| 0.8 | 0.7881 | 2.1 | 0.9821 | 3.4 | 0.99966 |
| 0.9 | 0.8159 | 2.2 | 0.9861 | 3.5 | 0.99977 |
| 1.0 | 0.8413 | 2.3 | 0.9893 | 3.6 | 0.99984 |
| 1.1 | 0.8643 | 2.4 | 0.9918 | 3.7 | 0.99989 |
| 1.2 | 0.8849 | 2.5 | 0.9938 | 3.8 | 0.99993 |

(*Contd.*)

(*b*) *For x in* 0.01 *intervals*

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|
| 1.60 | 0.9452 | 1.87 | 0.9693 | 2.14 | 2.9838 |
| 1.61 | 0.9463 | 1.88 | 0.9699 | 2.15 | 0.9842 |
| 1.62 | 0.9474 | 1.89 | 0.9706 | 2.16 | 0.9846 |
| 1.63 | 0.9484 | 1.90 | 0.9713 | 2.17 | 0.9850 |
| 1.64 | 0.9495 | 1.91 | 0.9719 | 2.18 | 0.9854 |
| 1.65 | 0.9505 | 1.92 | 0.9726 | 2.19 | 0.9857 |
| 1.66 | 0.9515 | 1.93 | 0.9732 | 2.20 | 0.9861 |
| 1.67 | 0.9525 | 1.94 | 0.9738 | 2.21 | 0.9865 |
| 1.68 | 0.9535 | 1.95 | 0.9744 | 2.22 | 0.9868 |
| 1.69 | 0.9545 | 1.96 | 0.9750 | 2.23 | 0.9871 |
| 1.70 | 0.9554 | 1.97 | 0.9756 | 2.24 | 0.9875 |
| 1.71 | 0.9564 | 1.98 | 0.9761 | 2.25 | 0.9878 |
| 1.72 | 0.9573 | 1.99 | 0.9767 | 2.26 | 0.9881 |
| 1.73 | 0.9582 | 2.00 | 0.9772 | 2.27 | 0.9884 |
| 1.74 | 0.9591 | 2.01 | 0.9778 | 2.28 | 0.9887 |
| 1.75 | 0.9599 | 2.02 | 0.9783 | 2.29 | 0.9890 |
| 1.76 | 0.9608 | 2.03 | 0.9788 | 2.30 | 0.9893 |
| 1.77 | 0.9616 | 2.04 | 0.9793 | 2.31 | 0.9896 |
| 1.78 | 0.9625 | 2.05 | 0.9798 | 2.32 | 0.9898 |
| 1.79 | 0.9633 | 2.06 | 0.9803 | 2.33 | 0.9901 |
| 1.80 | 0.9641 | 2.07 | 0.9808 | 2.34 | 0.9904 |
| 1.81 | 0.9649 | 2.08 | 0.9812 | 2.35 | 0.9906 |
| 1.82 | 0.9656 | 2.09 | 0.9817 | 2.36 | 0.9909 |
| 1.83 | 0.9664 | 2.10 | 0.9821 | 2.37 | 0.9911 |
| 1.84 | 0.9671 | 2.11 | 0.9826 | 2.38 | 0.9913 |
| 1.85 | 0.9678 | 2.12 | 0.9830 | 2.39 | 0.9916 |
| 1.86 | 0.9686 | 2.13 | 0.9834 | 2.40 | 0.9918 |

The function tabulated is

$$\Pr(x < X) = \int_{-\infty}^{x} \frac{1}{\sqrt{(2\pi)}} e^{-(1/2)x^2}\, dx = \Phi(X)$$

Table C.2 *Percentiles of the standard normal distribution*

| $P$ | $x$ | $P$ | $x$ | $P$ | $x$ |
|---|---|---|---|---|---|
| 20 | 0.8416 | 5 | 1.6449 | 2 | 2.0537 |
| 15 | 1.0364 | 4 | 1.7507 | 1 | 2.3263 |
| 10 | 1.2816 | 3 | 1.8808 | 0.5 | 2.5758 |
| 6 | 1.5548 | 2.5 | 1.9600 | 0.1 | 3.0902 |

This table gives one-sided percentage points,

$$P/100 = \int_{x}^{\infty} \frac{1}{\sqrt{(2\pi)}} e^{-(1/2)x^2}\, dx$$

The two-sided percentage appropriate to any $x$ is $2P$.



Table C.3 *Percentage points of the*

| | *Probability in per cent* 20 | 10 |
|---|---|---|
| 1 | 3.08 | 6.31 |
| 2 | 1.89 | 2.92 |
| 3 | 1.64 | 2.35 |
| 4 | 1.53 | 2.13 |
| 5 | 1.48 | 2.01 |
| 6 | 1.44 | 1.94 |
| 7 | 1.42 | 1.89 |
| 8 | 1.40 | 1.86 |
| 9 | 1.38 | 1.83 |
| 10 | 1.37 | 1.81 |
| 11 | 1.36 | 1.80 |
| 12 | 1.36 | 1.78 |
| 13 | 1.35 | 1.77 |
| 14 | 1.34 | 1.76 |
| 15 | 1.34 | 1.75 |
| 20 | 1.32 | 1.72 |
| 25 | 1.32 | 1.71 |
| 30 | 1.31 | 1.70 |
| 40 | 1.30 | 1.68 |
| 60 | 1.30 | 1.67 |
| 120 | 1.29 | 1.66 |
| ∞ | 1.28 | 1.64 |

This table gives two-sided pe

$P/$

where $f(x|\nu)$ is the p.d.f. of th
For one-sided percentage p

Table C.3 *Percentage points of the t-distribution*

| | *Probability in per cent* 20 | 10 | 5 | 2 | 1 | 0.1 |
|---|---|---|---|---|---|---|
| 1 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 | 31.60 |
| 3 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 | 12.92 |
| 4 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 1.48 | 2.01 | 2.57 | 3.36 | 4.03 | 6.87 |
| 6 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 1.42 | 1.89 | 2.36 | 3.00 | 3.50 | 5.41 |
| 8 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 | 4.32 |
| 13 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 | 4.22 |
| 14 | 1.34 | 1.76 | 2.14 | 2.62 | 2.98 | 4.14 |
| 15 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 20 | 1.32 | 1.72 | 2.09 | 2.53 | 2.85 | 3.85 |
| 25 | 1.32 | 1.71 | 2.06 | 2.48 | 2.79 | 3.72 |
| 30 | 1.31 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| 40 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 | 3.55 |
| 60 | 1.30 | 1.67 | 2.00 | 2.39 | 2.66 | 3.46 |
| 120 | 1.29 | 1.66 | 1.98 | 2.36 | 2.62 | 3.37 |
| ∞ | 1.28 | 1.64 | 1.96 | 2.33 | 2.58 | 3.29 |

This table gives two-sided percentage points,

$$P/100 = 2\int_t^{\infty} f(x|v)\,dx$$

where $f(x|v)$ is the p.d.f. of the $t$-distribution.

For one-sided percentage points the percentages shown should be halved.

Table C.4 *Percentage points of the $\chi^2$-distribution*

| *Degrees of freedom* ($v$) | 1 | 5 | 90 | 95 | 99 | 99.9 |
|---|---|---|---|---|---|---|
| | 0.000157 | | | | | |
| 1 | 0.03157 | 0.00393 | 2.71 | 3.84 | 6.63 | 10.83 |
| 2 | 0.0201 | 0.103 | 4.61 | 5.99 | 9.21 | 13.81 |
| 3 | 0.115 | 0.352 | 6.25 | 7.81 | 11.34 | 16.27 |
| 4 | 0.297 | 0.711 | 7.78 | 9.49 | 13.28 | 18.47 |
| 5 | 0.554 | 1.15 | 9.24 | 11.07 | 15.09 | 20.52 |
| 6 | 0.872 | 1.64 | 10.64 | 12.59 | 16.81 | 22.46 |
| 7 | 1.24 | 2.17 | 12.02 | 14.07 | 18.48 | 24.32 |
| 8 | 1.65 | 2.73 | 13.36 | 15.51 | 20.09 | 26.12 |
| 9 | 2.09 | 3.33 | 14.68 | 16.92 | 21.67 | 27.88 |
| 10 | 2.56 | 3.94 | 15.99 | 18.31 | 23.21 | 29.59 |
| 11 | 3.05 | 4.57 | 17.28 | 19.68 | 24.73 | 31.26 |
| 12 | 3.57 | 5.23 | 18.55 | 21.03 | 26.22 | 32.91 |
| 14 | 4.66 | 6.57 | 21.06 | 23.68 | 29.14 | 36.12 |
| 16 | 5.81 | 7.96 | 23.54 | 26.30 | 32.00 | 39.25 |
| 18 | 7.01 | 9.39 | 25.99 | 28.87 | 34.81 | 42.31 |
| 20 | 8.26 | 10.85 | 28.41 | 31.41 | 37.57 | 45.31 |
| 22 | 9.54 | 12.34 | 30.81 | 33.92 | 40.29 | 48.27 |
| 24 | 10.86 | 13.85 | 33.20 | 36.42 | 42.98 | 51.18 |
| 26 | 12.20 | 15.38 | 35.56 | 38.89 | 45.64 | 54.05 |
| 28 | 13.56 | 16.93 | 37.92 | 41.34 | 48.28 | 56.89 |
| 30 | 14.95 | 18.49 | 40.26 | 43.77 | 50.89 | 59.70 |

The table gives the percentage points $\chi^2$, where

$$P = 100 \int_0^{x^2} g(y|v)\,dy$$

where $g(y|v)$ is the probability density function of the $\chi^2$-distribution.

For $v > 30$, $\sqrt{(2\chi^2)}$ is approximately normally distributed with mean $(2v - 1)$ and unit variance.



Table C.5 *Percentage points of the*
(*a*) 95% points

| | *Degrees of freedom of* | 1 | 2 | 3 |
|---|---|---|---|---|
| *Degrees of freedom of denominator* ($v_2$) | 1 | 161.4 | 199.5 | 215.7 |
| | 2 | 18.51 | 19.00 | 19.1 |
| | 3 | 10.13 | 9.55 | 9.2 |
| | 4 | 7.71 | 6.94 | 6.5 |
| | 5 | 6.61 | 5.79 | 5.4 |
| | 6 | 5.99 | 5.14 | 4.7 |
| | 7 | 5.59 | 4.74 | 4.3 |
| | 8 | 5.32 | 4.46 | 4.0 |
| | 9 | 5.12 | 4.26 | 3.8 |
| | 10 | 4.96 | 4.10 | 3.7 |
| | 11 | 4.84 | 3.98 | 3.5 |
| | 12 | 4.75 | 3.89 | 3.4 |
| | 14 | 4.60 | 3.74 | 3.3 |
| | 16 | 4.49 | 3.63 | 3.2 |
| | 18 | 4.41 | 3.55 | 3.1 |
| | 20 | 4.35 | 3.49 | 3.1 |
| | 25 | 4.24 | 3.39 | 2.9 |
| | 30 | 4.17 | 3.32 | 2.9 |
| | 40 | 4.08 | 3.23 | 2.8 |
| | 60 | 4.00 | 3.15 | 2.7 |
| | ∞ | 3.84 | 3.00 | 2.6 |

(*b*) 97.5% points

| | *Degrees of freedom c* | 1 | 2 | 3 |
|---|---|---|---|---|
| *Degrees of freedom of denominator* ($v_2$) | 1 | 648 | 800 | 864 |
| | 2 | 38.5 | 39.0 | 39.2 |
| | 3 | 17.4 | 16.0 | 15.4 |
| | 4 | 12.2 | 10.6 | 9.9 |
| | 5 | 10.0 | 8.43 | 7.7 |
| | 6 | 8.81 | 7.26 | 6.6 |
| | 7 | 8.07 | 6.54 | 5.8 |
| | 8 | 7.57 | 6.06 | 5.4 |
| | 9 | 7.21 | 5.71 | 5.0 |
| | 10 | 6.94 | 5.46 | 4.8 |
| | 11 | 6.72 | 5.26 | 4.6 |
| | 12 | 6.55 | 5.10 | 4.4 |
| | 14 | 6.30 | 4.86 | 4.2 |
| | 16 | 6.12 | 4.69 | 4.0 |
| | 18 | 5.98 | 4.56 | 3.9 |
| | 20 | 5.87 | 4.46 | 3.8 |
| | 25 | 5.69 | 4.29 | 3.6 |
| | 30 | 5.57 | 4.18 | 3.5 |
| | 40 | 5.42 | 4.05 | 3.4 |
| | 60 | 5.29 | 3.93 | 3.3 |
| | ∞ | 5.02 | 3.69 | 3.1 |