MIDTERM REVIEW

Midterm

- Thursday, Mar 7 in Rosenau 230 from 930a-1045a
- Closed book
- No notes except one-sided, hand-written, 8.5×11 page of notes
- Bring calculator, but no smart phones, laptops, tablets, etc
- Blank/scratch paper

Midterm

- Will cover HR §1 – 10, except §5.4 – 5.6
- Recommended to review (in order of priority)
  1. Notes
  2. HW
  3. HR
- Qs may be directly from the HW or very similar to HW probs; probs with lengthy answers are unlikely to be on the midterm, whereas simpler problems are more likely
- Topics from HR not in the notes or HW less likely to be on midterm

Definition of Causal Effect (§1)

- Potential outcomes/counterfactuals
- Causal consistency

$$Y = Y^{a=1}A + Y^{a=0}(1-A)$$

- SUTVA: no interference, not multiple versions of treatment
- Measures of causal effect vs measures of association, eg,

$$E(Y^{a=1}) - E(Y^{a=0})$$

vs

$$E(Y|A=1) - E(Y|A=0)$$

Randomized Experiments (§2)

- Full exchangeability

$$\{Y^{a=0}, Y^{a=1}\} \perp\!\!\!\perp A$$

implies exchangeability

$$Y^a \perp\!\!\!\perp A \text{ for } a = 0, 1$$

implies mean exchangeability

$$E[Y^a | A = 1] = E[Y^a | A = 0] \text{ for } a = 0, 1$$

- Under mean exchangeability

$$E[Y^a] = E[Y | A = a],$$

implying causal measures identifiable

Randomized Experiments (§2)

- Conditionally randomized experiments: conditional exchangeability

$$Y^a \perp\!\!\!\perp A | L \text{ for } a = 0, 1$$

- Standardization. Under conditional exchangeability

$$E[Y^a] = \sum_l E[Y|A = a, L = l]\,\Pr[L = l]$$

suggesting estimators of the form

$$\widehat{RR} = \frac{\sum_l \widehat{\Pr}[Y = 1|L = l, A = 1]\widehat{\Pr}[L = l]}{\sum_l \widehat{\Pr}[Y = 1|L = l, A = 0]\widehat{\Pr}[L = l]}$$

- IPW

$$\widehat{E}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} \frac{I[A_i = a]Y_i}{\Pr[A_i = a|L_i]}$$

Observational Studies (§3)

- Randomized Experiment Paradigm: An observational study can be conceptualized as a conditionally randomized experiment under the following three conditions:

  1. Values of treatment under comparison correspond to well-defined interventions (§3.4–3.5)

  2. Conditional probability of receiving every value of treatment, though not decided by investigators, depends only on the measured covariates (§3.2)

  $$Y^a \perp\!\!\!\perp A | L \text{ for } a = 0, 1$$

  3. Conditional probability of receiving every value of treatment is positive (§3.3), i.e., for $a = 0, 1$

  $$\Pr[A = a | L = l] > 0 \text{ for all } l \text{ such that } \Pr[L = l] > 0$$

Effect Modification (§4)

- Let M be some baseline covariate (and thus not affected by treatment A) taking on values 0, 1
- The concept of effect modification is scale dependent
- There is additive effect modification if

$$E[Y^{a=1} - Y^{a=0}|M = 1] \neq E[Y^{a=1} - Y^{a=0}|M = 0]$$

- There is multiplicative effect modification if

$$\frac{E[Y^{a=1}|M = 1]}{E[Y^{a=0}|M = 1]} \neq \frac{E[Y^{a=1}|M = 0]}{E[Y^{a=0}|M = 0]}$$

Interaction (§5)

- Consider two possible interventions A and E such that each individual has four potential outcomes $Y^{a,e}$ for $a, e = 0, 1$

- *Interaction* between two treatments A and E if the causal effect of A on Y after a joint intervention that sets E to 1 differs from the causal effect of A on Y after a joint intervention that sets E to 0

- Interaction between A and E on the additive scale if

$$E[Y^{a=1,e=1} - Y^{a=0,e=1}] \neq E[Y^{a=1,e=0} - Y^{a=0,e=0}]$$

- Marginal structural model (MSM)

$$E[Y^{a,e}] = \beta_0 + \beta_1 a + \beta_2 e + \beta_3 ae$$

  Additive interaction iff $\beta_3 \neq 0$

Graphical Representation of Causal Effects (§6)

- Markov-factorization: Density (or pmf) $f(V)$ of variables V in DAG G satisfies the Markov factorization

$$f(v) = \prod_{j=1}^{M} f(v_j|pa_j)$$

Conditional on its parents, V_j is independent of its non-descendants

- Causal DAGs

1. Lack of arrow from V_j to V_m can be interpreted as the absence of a direct causal effect of V_j on V_m (relative to the other variables on the graph)

2. All common causes, even if unmeasured, of any pair of variables on the graph are themselves on the graph

3. Any variable is a cause of its descendants

Graphical Representation of Causal Effects (§6)

- D-separation
- A path is blocked iff

  (i) it contains a noncolllider that has been conditioned on, or

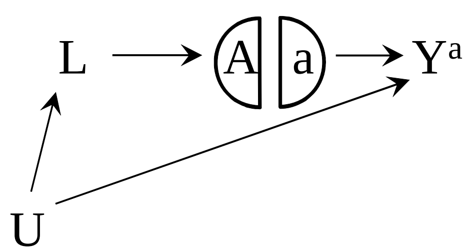
  (ii) it contains a collider which has not been conditioned on and has no descendant that has been conditioned on
- Two variables are d-separated if all paths between them are blocked

  Otherwise the two variables are d-connected
- If two variables (e.g., A and Y) are *d-separated* given some other variable (e.g., L), then then the two variables are conditionally independent given the third $A \perp\!\!\!\perp Y | L$

$$\begin{tikzcd} \boxed{\mathrm{L}} \arrow[r] \arrow[rr, bend left] & \mathrm{A} & \mathrm{Y} \end{tikzcd}$$

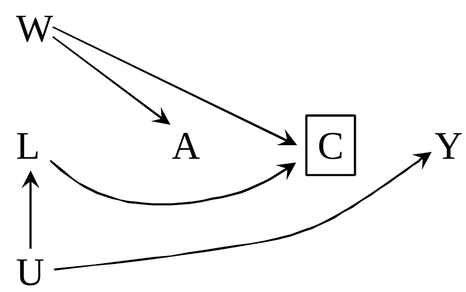
Confounding Bias (§7)

- Confounding is the bias that arises when the treatment and the outcome share a common cause
- Backdoor criterion: effect of treatment A on the outcome Y is identifiable if all backdoor paths between them can be blocked by conditioning on some set of measured variables which are non-descendants A
- SWIGs
  1. Split intervention node or nodes
  2. Replace all descendants of split nodes with potential outcomes
- Eg Fig 7.7



Selection Bias (§8)

- General definition of selection bias: Conditioning on a common effect of (i) treatment A or a cause of A, and (ii) outcome Y or a cause of Y



- Differential loss-to-follow-up, informative censoring, missing data bias, nonresponse bias (of complete case analysis), healthy worker bias, self-selection bias, volunteer bias, case-control studies

- Under certain assumptions can account for selection bias using IPW or standardization, e.g.,

$$\hat{E}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} \frac{I[A_i = a, C_i = 0]Y_i}{\Pr[C_i = 0|A_i, L_i]\Pr[A_i = a|L_i]}$$

Measurement Bias (§9)

- *Measurement bias* when the association between treatment and outcome is weakened or strengthened as a result of the process by which the study data are measured

- Measurement error in A or Y
  Dependent/independent
  Differential/non-differential

- Confounders L may also be measured with error; can cause bias even if A and Y measured without error

$$\begin{tikzcd} \mathrm{L}^* & & \\ \mathrm{L} \arrow[u] \arrow[r] \arrow[rr, bend left] & \mathrm{A} \arrow[r] & \mathrm{Y} \end{tikzcd}$$

- Non-compliance in randomized trials:
  ITT, per-protocol, compliers

Random Variability (§10)

- Large sample frequentist inference

  Assume $(Y_1, A_1), \ldots, (Y_n, A_n)$ i.i.d. based on a random sample from an infinite (super-)population

  Under exchangeability, i.e., $Y^a \perp\!\!\!\perp A$,

$$\frac{\sum_i Y_i I(A_i = a)}{\sum_i I(A_i = a)} \xrightarrow{p} E(Y^a)$$

- Randomization-based inference

  Sharp null hypothesis, Fisher's exact test, permutation test, etc