

Overview

- Key characteristic: censoring

Right (type I/II), left, interval

T : Potential Failure Time (r.v., $T \geq 0$)

C : Potential Censoring Time

$X = \min(T, C)$

δ : 0 or 1 r.v. $\left\{ \begin{array}{l} = 1 \text{ if failed} \\ = 0 \text{ if censored} \end{array} \right.$

- Methods

Distribution of failure times

Comparison of failure times

Association of covariates with survival using parametric models

Fundamentals

1. Survival function $S(t) = \Pr[T > t]$

2. Hazard function

$$h(t) = \lim_{\Delta t \rightarrow 0^+} \frac{P(t \leq T < t + \Delta t | T \geq t)}{\Delta t}$$

3. Probability density function (PDF)

$$f(t) = \lim_{\Delta t \rightarrow 0^+} \frac{1}{\Delta t} P[t \leq T < t + \Delta t]$$

4. Cumulative distribution function (CDF)

$$F(t) = P[T \leq t] = \int_0^t f(s) ds \quad 1 - F(t) = P(T > t) = S(t)$$

5. Cumulative hazard function

$$H(t) = \int_0^t h(s) ds = -\log(S(t))$$

Summary of the relationships among functions 1-5

$$F(t) = \int_0^t f(s)ds = 1 - S(t)$$

$$S(t) = 1 - F(t) = \int_t^\infty f(s)ds = e^{-\int_0^t h(s)ds} = e^{-H(t)}$$

$$h(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1 - F(t)} = \frac{\frac{dF(t)}{dt}}{1 - F(t)} = -\frac{\frac{dS(t)}{dt}}{S(t)}$$

$$H(t) = \int_0^t h(s)ds = -\log(S(t)) = -\log(1 - F(t))$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dS(t)}{dt} = h(t)S(t) = h(t)(1 - F(t))$$

- Distributions
 - Exponential
 - Weibull
 - Log-logistic
 - Log-normal
 - Gamma
- Large sample theory eg MLE

One, Two Sample Problems

S: Kaplan-Meier estimator (Greenwood SE), Life-table estimator

H: Nelson-Aalen

h: Kernel-Smoother

H_0 : Logrank test, Gehan-Breslow, Peto-Peto; Z

Stratified, G sample, trend

Parametric Models

AFT

$$S(t|Z = z) = S_0(t/\phi(z)) \text{ where } \phi(z) = \exp(\beta z)$$

$$\log(T|Z = z) = \mu + \beta z + \sigma \epsilon$$

Weibull, Log-logistic, Log-normal; Fitting, interpretation, diagnostics, etc

PH

$$h(t|Z = z) = h_0(t) \exp(\beta_{PH} z)$$

Weibull

P0

$$S(t|Z = z)/\{1 - S(t|Z = z)\} = S_0(t)/\{1 - S_0(t)\} \exp(\beta_{P0} z)$$

Log-logistic

Midterm

Review notes, HW; Read Collett 1, 2, 5, 6, 7, Appendix A

Simple calculations by hand

Interpret SAS/R output

Proof

Thursday 3/8: 8.5x11 one-sided hand-written notes; calculator; blank paper