Power and Sample Size III
Bios 662

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Power and Sample Size

- Many of the sample size/power formulae assume a balanced design (exception: case-control)
- How do we generalize to unbalanced designs?
- For example, consider two-sample test with continuous outcome

Two-sample t-test

- Assume normality, HOV

$$\bar{Y}_i \sim N(\mu_i, \sigma^2/N_i) \text{ for } i = 1, 2$$

- Under $H_0 : \mu_1 - \mu_2 = 0$

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{s_p\sqrt{1/N_1 + 1/N_2}} \sim t_{N_1+N_2-2}$$

- Under $H_A : \mu_2 - \mu_2 = \delta_A > 0$

$$\bar{Y}_1 - \bar{Y}_2 \sim N(\delta_A, \sigma^2(1/N_1 + 1/N_2))$$

implying

$$\frac{\bar{Y}_1 - \bar{Y}_2}{\sigma\sqrt{1/N_1 + 1/N_2}} \sim N\left(\frac{\delta_A}{\sigma\sqrt{1/N_1 + 1/N_2}}, 1\right)$$

Two-sample t-test

- Note

$$\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{\sigma^2} \sim \chi^2_{N_1+N_2-2}$$

- Therefore

$$\frac{(\bar{Y}_1-\bar{Y}_2)/(\sigma\sqrt{1/N_1+1/N_2})}{\sqrt{\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{\sigma^2(N_1+N_2-2)}}} \sim t_{N_1+N_2-2,\delta_A/(\sigma\sqrt{1/N_1+1/N_2})}$$

- Equivalently

$$\frac{\bar{Y}_1-\bar{Y}_2}{s_p\sqrt{1/N_1+1/N_2}} \sim t_{N_1+N_2-2,\delta_A/(\sigma\sqrt{1/N_1+1/N_2})}$$

Two-sample t-test

- Given N_1, N_2, α, δ_A, power equals $\Pr[T > t_{1-\alpha/2}]$ where $T \sim t_{N_1+N_2-2,\delta_A/(\sigma\sqrt{1/N_1+1/N_2})}$
- For example, suppose $N_1 = 10$, $N_2 = 20$, $\alpha = 0.05$, $\delta_A = 15$, $\sigma = 25$.
- R

```
> 1-pt(qt(.975,28),28,15/(25*sqrt(1/10+1/20)))
[1] 0.3214083
```

- SAS

```
proc power;
   twosamplemeans
   meandiff   = 15
     groupns = 10|20
     stddev = 25
     power  = .;
run;
```