Solutions to Problem Set 2

4.6.1 Without loss of generality, assume $u \leq v$. Then

$$
\begin{aligned}
|\eta(u)-\eta(v)| &= \left|\int_u^v \frac{\dot\eta(e)}{\eta(e)}\eta(e)de\right| \\
&= \left|\int_{-\infty}^{\infty} \mathbf{1}\{u \leq s \leq v\}\frac{\dot\eta(e)}{\eta(e)}\eta(e)de\right| \\
&\leq \left(\int_{-\infty}^{\infty} \mathbf{1}\{u \leq s \leq v\}\eta(e)de\right)^{1/2}\left(\int_{-\infty}^{\infty}\left[\frac{\dot\eta(e)}{\eta(e)}\right]^2\eta(e)de\right)^{1/2} \\
&= |F(u)-F(v)|^{1/2}\left(P\left[\frac{\dot\eta(e)}{\eta(e)}\right]^2\right)^{1/2}. \Box
\end{aligned}
$$

4.6.8 Since $E[Y|Z=z,U=u]=\mu_{\beta,\eta}(u,z)$, both $(Z-h_1(U))(Y-\mu_{\beta,\eta}(Z,U))$ and $h(U)(Y-\mu_{\beta,\eta}(Z,U))$ have mean zero. Thus it is sufficient to verify

$$
E\left[(Z-h_1(U))h(U)(Y-\mu_{\beta,\eta}(Z,U))^2\right] = 0. \tag{1}
$$

Note by definition that

$$
V_{\beta,\eta}(z,u)=E\left[(Y-\mu_{\beta,\eta}(Z,U))^2|Z=z,U=u\right].
$$

Thus we have

$$
\begin{aligned}
E\left[h_1(U)h(U)(Y-\mu_{\beta,\eta}(Z,U))^2|U=u\right] &= h_1(u)h(u)E\left[V_{\beta,\eta}(Z,U)|U=u\right], \\
&= h(u)E\left[ZV_{\beta,\eta}(Z,U)|U=u\right],
\end{aligned}
$$

by definition of $h_1(U)$. Since also

$$
E\left[Zh(U)(Y-\mu_{\beta,\eta}(Z,U))^2|U=u\right]=h(u)E\left[ZV_{\beta,\eta}(Z,U)|U=u\right],
$$

the desired result (1) follows.$\Box$