Advanced Probability and Statistical Inference I (BIOS 760)
Fall 2017

- **COURSE DESCRIPTION** (4 credit hours)

  The course introduces fundamental concepts of measure theory and probability measure theory. Large sample theory in probability measure spaces is given, including a variety of convergence results and central limit theorems. The second part of the course reviews a number of methods for point estimation and statistical prediction, with particular attention to maximum likelihood estimation, semiparametric methods, machine learning, and related areas.

- **CLASS TIME**: 1:25 A.M.–3:10 P.M., Mondays and Wednesdays in Michael Hooker Building room HC 0001

- **CLASS WEBSITE**: `http:\\www.bios.unc.edu\~kosorok\BIOS760.html`
  (Contains assignments, due dates, exam information, etc.)

- **TEXTS FOR TEACHING**

    - Lecture notes (downloadable from the website)

    - *Theory of Point Estimation*, Second Edition, Lehmann, E. and Casella, G., 1998

    - *A Course in Large Sample Theory*, Ferguson, T. S., 1996, reprinted 2002

- **INSTRUCTOR**

  Dr. Michael R. Kosorok (Email: kosorok@unc.edu)

  Phone: 919-966-8107

  Office: 3101 McG

  Office hours: 3:30–5:30 PM Mondays

- **GRADER**

  Daniel Luckett (Email: luckett@live.unc.edu)

  Office: 2305 McG

  Office hours: 3:30-4:30 PM Tuesdays

- **GRADING SYSTEM**

  Final grade is based on performance on weekly homework, two midterm exams and one

final exam. The distribution is respectively 40%, 20%, 20% and 20%. The grades reported will be transformed to an "HPF" scale (H: 85–100; P: 70–84; LP: 60–69; F: 0–59).

- **MAIN TOPICS**

  1. Distribution Theory (expected 1 week)

     – Basic concepts

     – Special distributions

     – Algebra and transformation of random variables

     – Multivariate normal distribution

     – Families of distributions

  2. Measure, Integration and Probability (expected 3 weeks)

     – Set theory and topology

     – Measure space

     – Construction of measure space

     – Measurable functions and integration

     – Product of measures–Fubini-Tonelli Theorem

     – Derivative of measures–Radon-Nikodym Theorem

     – Probability measure

     – Conditional probability and independence

  3. Large Sample Theory of Random Variables (expected 4 weeks)

     – Modes of convergence

     – Convergence in distribution

     – Limit theorems for summation of independent random variables

     – Limit theorems for summation of non-independent random variables–U-statistics and Martingales

     – Some notation

  4. Point Estimation and Efficiency (expected 2 weeks)

     – Methods of point estimation

     – Crámer-Rao bound for parametric models

     – Information bound and efficient influence function

      – Asymptotic efficiency bound: Le Cam's lemmas

5. Efficient Estimation: Maximum Likelihood Approach (expected 3 weeks)

      – Kullback-Leibler information

      – Consistency of maximum likelihood estimators

      – Asymptotic efficiency of maximum likelihood estimators

      – Computation of maximum likelihood estimators: The EM algorithm

      – Nonparametric maximum likelihood estimation

6. Other Topics, Including Machine Learning

- **OTHER INFORMATION**

  – A number of problems are given at the end of each chapter of the lecture notes. Homework will be assigned from these problems and solutions will be posted in the class webpage after grading. You are encouraged to work on the problems not assigned. Working in groups is not discouraged but plagiarism (copying) is strictly prohibited. It is recommended that, if you wish to work in groups, that your first try the problems on your own before discussing them with the group. You may use ideas you obtain from this group interaction for your solutions, but you must process the solutions and write them in your own words. As stated before, copying problem solutions is strictly prohibited, and, moreover, it is a violation of the honor code.

  – Teaching tools will be mainly based on the use of the projector, sometimes with the help of the chalkboard or handouts. The slides for teaching can be downloaded from the webpage. You may wish to print out the slides with the blank note pages so that additional notes can be taken side by side.

  – The two midterms and the final exam will be closed-book exams. The midterms will be held during regular class slots.

  – Feel free to send me an email or stop by during my office hours if you have questions, comments, ideas or suggestions.

  – Work hard and never give up!