

Advanced Probability and Statistical Inference I (BIOS 760)

Fall 2008

- **COURSE DESCRIPTION** (4 credit hours)

The course introduces fundamental concepts of measure theory and probability measure theory. Large sample theory in probability measure space is given, including a variety of convergence results and the central limit theorems. The second part of the course reviews a number of methods for point estimation, with particular attention to maximum likelihood estimation and its related aspects.

- **MEETING TIME:** 3:30–5:15 P.M., Tuesdays and Thursdays in room McG-G 1304

- **CLASS WEBSITE:** <http://www.bios.unc.edu/~kosorok/BIOS760.html>

- **TEXTS FOR TEACHING**

- Lecture notes (downloadable from the website)
- *Theory of Point Estimation*, Second Edition, Lehmann, E. and Casella, G., 1998
- *A Course in Large Sample Theory*, Ferguson, T. S., 1996, reprinted 2002

- **INSTRUCTOR**

Dr. Michael R. Kosorok

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Office hours: 1–2 PM Tuesday, 2–3 PM Thursday, and some additional time slots by appointment

- **TEACHING ASSISTANT**

Dr. Emil Cornea

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Office hours: 11:00AM–noon on Monday, in room MHRC 3100; and noon–1:00PM on Friday, in room MHRC 0003.

- **GRADING SYSTEM**

Final grade is based on the performance of bi-weekly homework, one midterm exam and

one final exam. The distribution is respectively 40%, 30% and 30%. The grades reported will be transformed to an “HPF” scale (H: 85–100; P: 70–84; LP: 60–69; F: 0–59).

• **MAIN TOPICS**

1. Distribution Theory (expected 1 week)
 - Basic concepts
 - Special distributions
 - Algebra and transformation of random variables
 - Multivariate normal distribution
 - Families of distributions
2. Measure, Integration and Probability (expected 3 weeks)
 - Set theory and topology
 - Measure space
 - Construction of measure space
 - Measurable functions and integration
 - Product of measures–Fubini-Tonelli Theorem
 - Derivative of measures–Radon-Nikodym Theorem
 - Probability measure
 - Conditional probability and independence
3. Large Sample Theory of Random Variables (expected 4 weeks)
 - Modes of convergence
 - Convergence in distribution
 - Limit theorems for summation of independent random variables
 - Limit theorems for summation of non-independent random variables–U-statistics and Martingales
 - Some notation
4. Point Estimation and Efficiency (expected 2 weeks)
 - Methods of point estimation
 - Crámer-Rao bound for parametric models
 - Information bound and efficient influence function

- Asymptotic efficiency bound: Le Cam's lemmas
- 5. Efficient Estimation: Maximum Likelihood Approach (expected 3 weeks)
 - Kullback-Leibler information
 - Consistency of maximum likelihood estimators
 - Asymptotic efficiency of maximum likelihood estimators
 - Computation of maximum likelihood estimators: EM algorithm
 - Nonparametric maximum likelihood estimation

• **OTHER INFORMATION**

- A number of problems are given at the end of each chapter of the lecture notes. Homework will be assigned from these problems and solutions will be posted in the class webpage after grading. You are encouraged to work on the problems not assigned. Working in groups is not discouraged but plagiarism (copying) is strictly prohibited. It is recommended that, if you wish to work in groups, that you first try the problems on your own before discussing them with the group. You may use ideas you obtain from this group interaction for your solutions, but you must process the solutions and write them in your own words. As stated before, copying problem solutions is strictly prohibited.
- Teaching tools will be mainly based on the use of the projector, sometimes with the help of the chalkboard or handouts. The slides for teaching can be downloaded from the webpage. You may wish to print out the slides with the blank note pages so that additional notes can be taken side by side.
- Both midterm and final exams will be closed-book exams. The midterm will be held during a regular class slot.
- Feel free to send me an email, give me a call or stop by during my office hours if you have questions, comments, ideas or suggestions.
- Work hard and never give up!