BIOS760: 2011 FALL SEMESTER MIDTERM EXAM II

1. Suppose that X_n and Y_n are independent random variables. Moreover, $X_n \to_d X$ and $Y_n \to_d Y$ for some random variables X and Y.

   (a) (3 points) Give one example that $X_n + Y_n$ does not converge to $X + Y$ in distribution.

   (b) (5 points) If we further assume that X and Y are independent, then show that $(X_n, Y_n) \to_d (X, Y)$. From this result, show that $X_n + Y_n \to_d X + Y$.

2. Let $Y_i = x_i\beta + \sqrt{x_i}\epsilon_i, i = 1, .., n,$, where $\epsilon_1, ..., \epsilon_n$ are i.i.d from some distribution with mean zero and variance 1, and x_i is a positive constant.

   (a) (3 points) Show that for any positive constants $(w_1, ..., w_n)$ such that $\sum_{i=1}^n w_i = 1$, $\sum_{i=1}^n w_i Y_i/x_i$ is an unbiased estimator for β.

   (b) (3 points) Find constants $(w_1, ..., w_n)$ to minimize the variance of the above estimator.

   (c) (6 points) Assume
   $$\frac{\max_{i=1}^n x_i}{\sum_{i=1}^n x_i} \to 0.$$
   Derive the asymptotic distribution of the estimator in (b) after proper normalization and justify your result.