1. (a) Note

\[ \text{Cov}(Z_1 - \rho Z_2, Z_2) = 0. \]

Thus, \( Z_1 - \rho Z_2 \) is independent of \( Z_2 \). Moreover, since \( Z_2 \sim N(0, \sigma^2) \) and

\[ Z_1 - \rho Z_2 \sim N(0, (1 - \rho^2)\sigma^2). \]

Hence,

\[ \frac{Z_2}{\sqrt{(Z_1 - \rho Z_2)^2}} \sim \frac{1}{\sqrt{1 - \rho^2}} t(1). \]

(b) The density is

\[ \exp \left\{ \frac{-Z_1^2}{2(1 - \rho^2)\sigma^2} + \frac{Z_1 Z_2 \rho}{(1 - \rho^2)\sigma^2} - \frac{Z_2^2}{2(1 - \rho^2)\sigma^2} - \log(\sqrt{1 - \rho^2}\sigma^2) \right\} / 2\pi. \]

It is a 2-parameter exponential family. Let

\[ \eta_1 = -\frac{1}{2(1 - \rho^2)\sigma^2}, \quad \eta_2 = \frac{\rho}{(1 - \rho^2)\sigma^2}. \]

Then the density can be rewritten as

\[ \exp \left\{ \eta_1 (Z_1^2 + Z_2^2) + \eta_2 Z_1 Z_2 + \frac{1}{2} \log(4\eta_1^2 - \eta_2^2) \right\} / 2\pi. \]

(c) Treating \( \eta_2 \) as the only parameter, we obtain the moment generating function for \( Z_1 Z_2 \) is

\[ \exp \left\{ \frac{-1}{2} \log(4\eta_1^2 - (\eta_2 + t)^2) + \frac{1}{2} \log(4\eta_1^2 - \eta_2^2) \right\} = \sqrt{\frac{4\eta_1^2 - \eta_2^2}{4\eta_1^2 - (\eta_2 + t)^2}}. \]

Its second derivative at \( t = 0 \) is

\[ (1 + 2\rho^2)\sigma^4. \]

Thus, \( E[Z_1^2 Z_2^2] \) is \( (1 + 2\rho^2)\sigma^4 \).

2. (a) Note \( \lambda_F(\{0\}) = 0 \) but \( \lambda_C(\{0\}) = 1 \) and \( \lambda_F((0,1]) = 1 - e^{-1} \) but \( \lambda_C((0,1]) = 0 \).

(b) Clearly \( \mu(\emptyset) = 0 \) and \( \mu(R) = 1 \). Since \( \lambda_F \) and \( \lambda_C \) both satisfy countable additivity, so is \( \mu \).
(c) For any \( x_0 > 0 \),
\[
\{ x : f(x) \leq x_0 \} = \{ x : x \leq 2 \log x_0 \} \in \mathcal{B};
\]
for any \( x_0 \leq 0 \), \( \{ x : f(x) \leq x_0 \} = \emptyset \in \mathcal{B} \). Thus, \( f(x) \) is a random variable.

\[
E[f] = \frac{1}{2} \int f(x) d\lambda_F(x) + \frac{1}{2} \int f(x) d\lambda_G(x)
\]
\[
= \frac{1}{2} \int e^{x/2} e^{-x} I(x \geq 0) dx + \frac{1}{2} f(0) = \frac{3}{2}.
\]

3. (a) For any \( \epsilon > 0 \),
\[
P(|X_n| > \epsilon) \leq P(|X| > n) \leq \frac{E[|X|]}{n} \to 0.
\]

(b) Since \( |X_n| \leq |X| \) and \( X_n \to_p 0 \), by DCT, \( E[X_n] \to 0 \).

(c) Since \( n|X_n| = n|X| I(|X| > n) \leq |X|^2 I(|X| > n) \leq |X|^2 \) and the same argument as in (a) gives \( nX_n \to_p 0 \), by DCT, \( nE[X_n] \to 0 \).