BIOS760: 2013 FALL SEMESTER FINAL EXAM

1. Let $X_1, ..., X_n$ be n i.i.d observations from Poisson(λ) where λ is an unknown and positive parameter.

   (a) (5 points) Find the UMVUE for e^{λ}.

   (b) (5 points) What is the information bound for e^{λ}? Does the UMVUE attain this bound?

   (c) (3 points) Derive the MLE for e^{λ} and its asymptotic distribution.

   (d) (2 points) What is the relative efficiency of the MLE with respect to the UMVUE, which is defined as the ratio between the variance of the UMVUE and the variance of the MLE?

2. We wish to observe n pairs of observations, $(Y_1, X_1), ..., (Y_n, X_n)$, from n i.i.d subjects. However, some subject's X_i's are missing and we use R_i to denote non-missingness of X_i ($R_i = 1$ indicates X_i observable; otherwise, $R_i = 0$). Then then the observed data can be written as

$$(Y_i, R_i X_i, R_i), \quad i = 1, ..., n.$$

   Assume

$$Y_i = \beta X_i + \epsilon_i, \quad X_i \sim N(0, 1), \quad \epsilon_i \sim N(0, \sigma^2),$$

   and ϵ_i and X_i are independent. Furthermore, the conditional probability of R_i given (Y_i, X_i) may depend on (Y_i, X_i).

   (a) (5 points) Write down the observed likelihood function. You must include the distribution of R given (Y, X) in the likelihood function.

   (b) (5 points) Assume that R_i only depends on Y_i. We treat the unobserved X's, i.e., those subjects with $R_i = 0$, as missing data. Write down the EM algorithm for estimating parameters (β, σ^2).

   (c) (5 points) If R_i depends on X_i and particularly, $R_i = I(X_i > 0)$, what is the EM algorithm for computing the maximum likelihood estimator for (β, σ^2)? You can leave the expressions in the algorithm.