BIOS 600 · Practice Quiz for Linear Regression

Fall 2011

1. In class, we discussed how residual diagnostics can help identify certain problems that affect the validity of a linear regression model. When we fit a linear regression model, we must check these diagnostics before making statements about X and Y or the βs. The next five pages show examples of datasets that

   (a) include outliers,

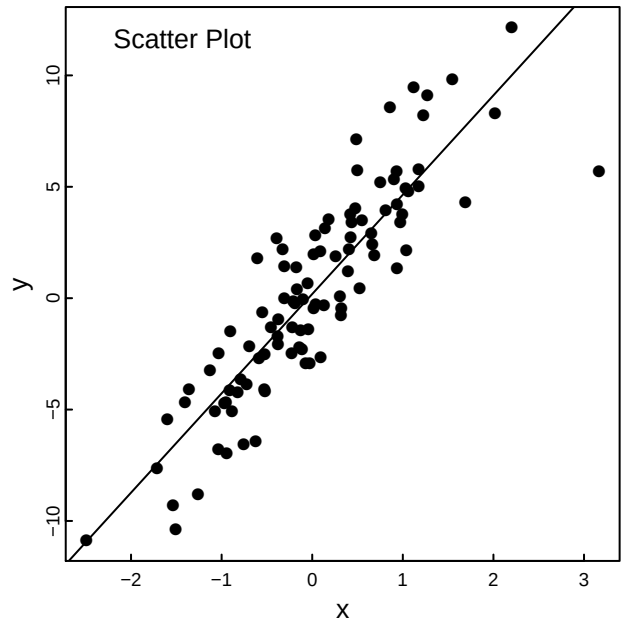
   (b) are heteroskedastic,

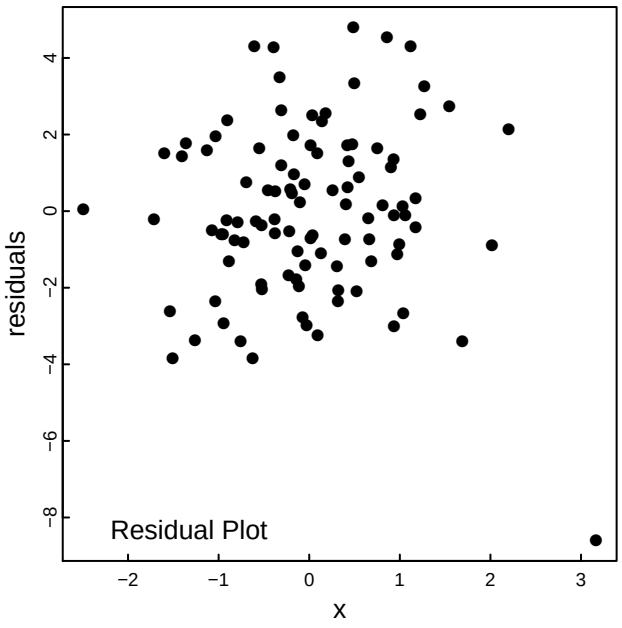
   (c) involve a non linear relationship,

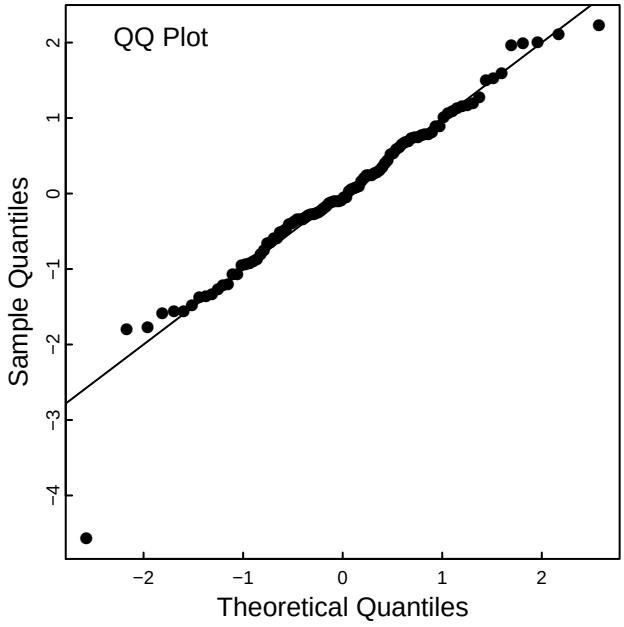
   (d) have non normal residuals, or

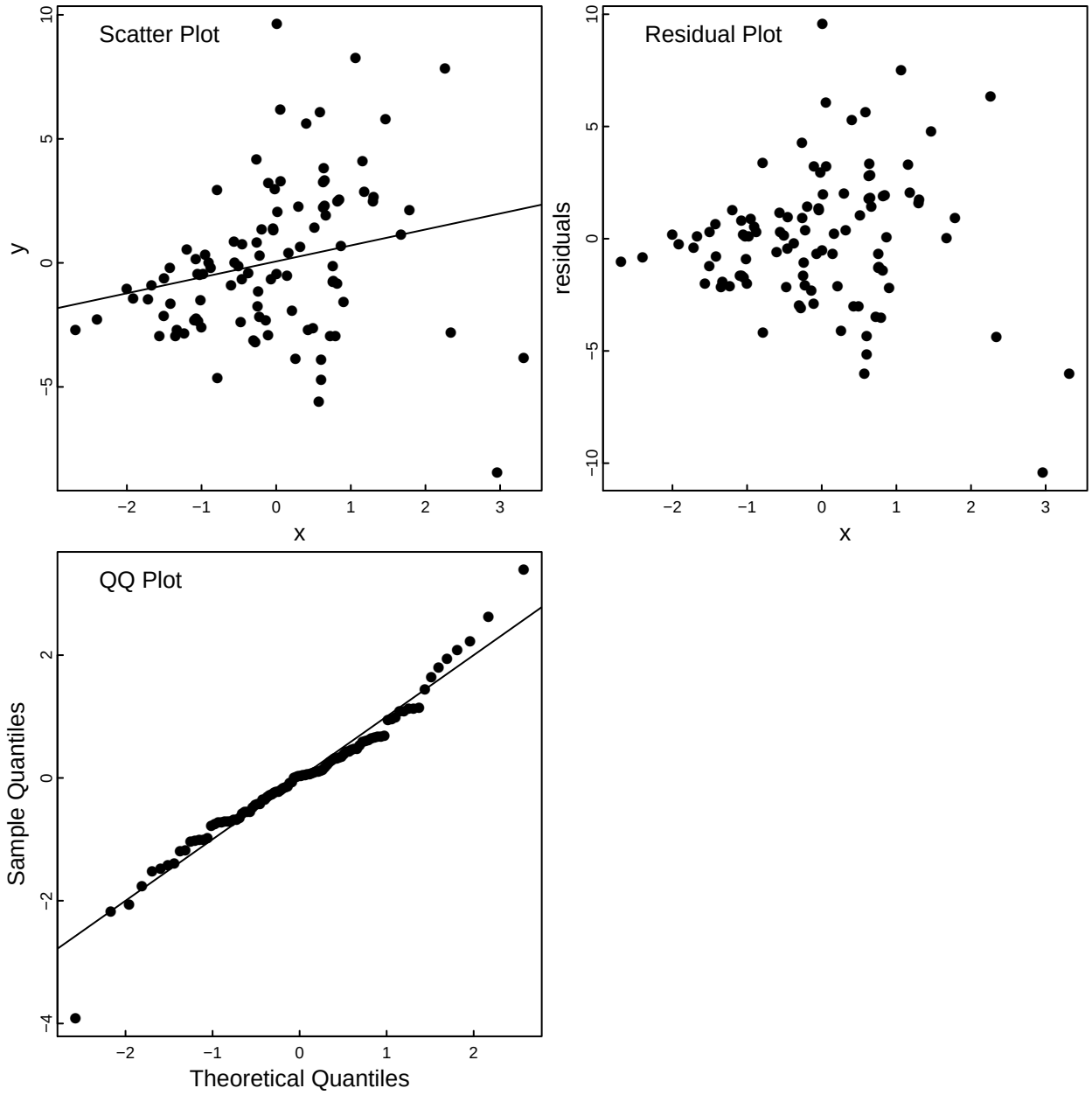
   (e) are without problems.

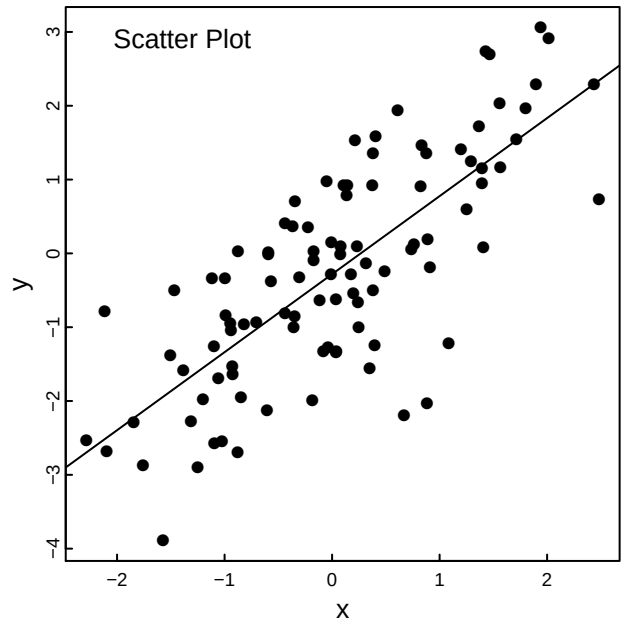
   Identify the dataset that corresponds to each one of these issues.

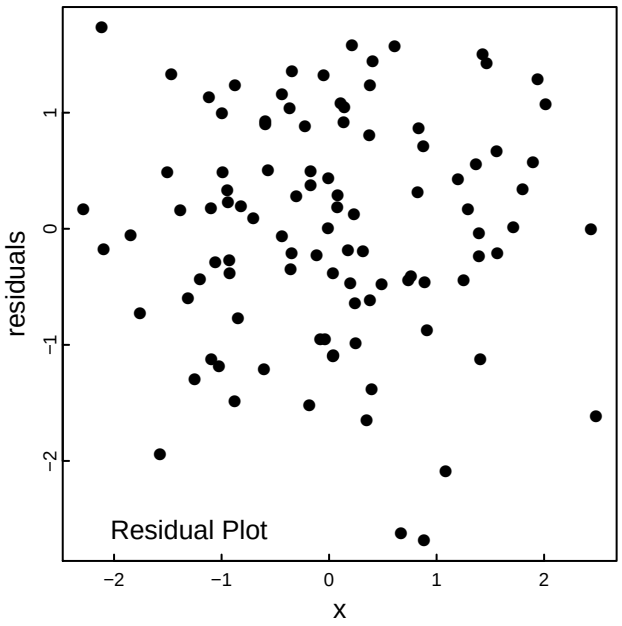

Scatter Plot
y
x
−10
−5
0
5
10
−2
−1
0
1
2
3

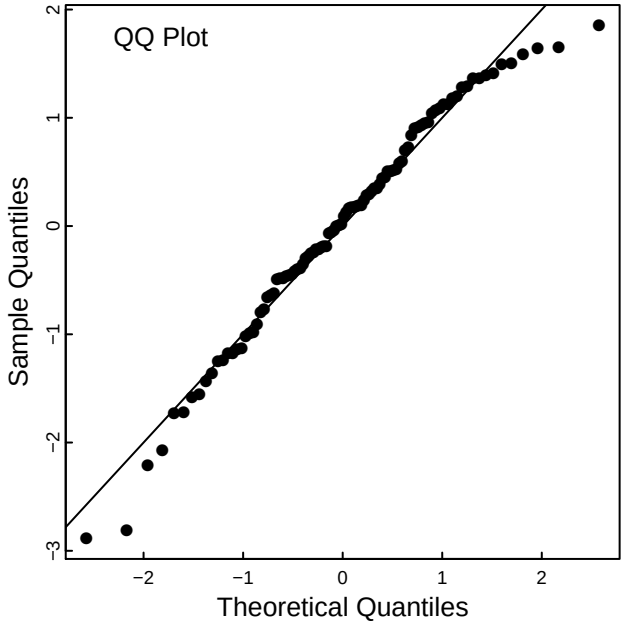

Residual Plot
residuals
x
−8
−6
−4
−2
0
2
4
−2
−1
0
1
2
3

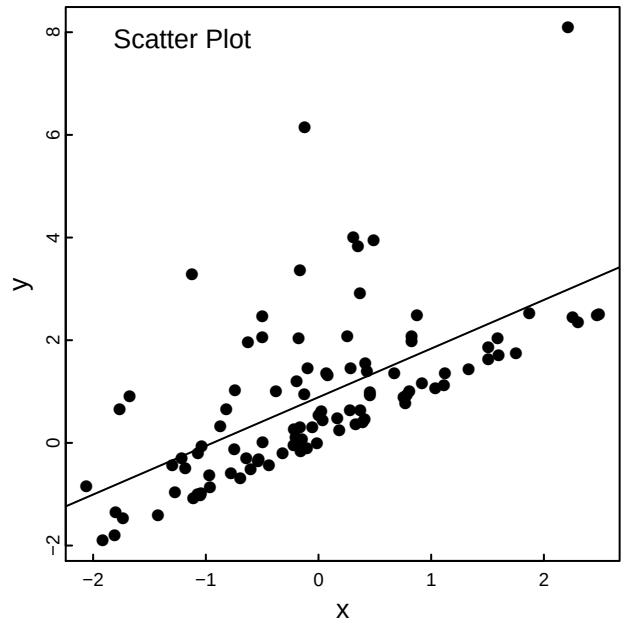

QQ Plot
Sample Quantiles
Theoretical Quantiles
−4
−3
−2
−1
0
1
2
−2
−1
0
1
2

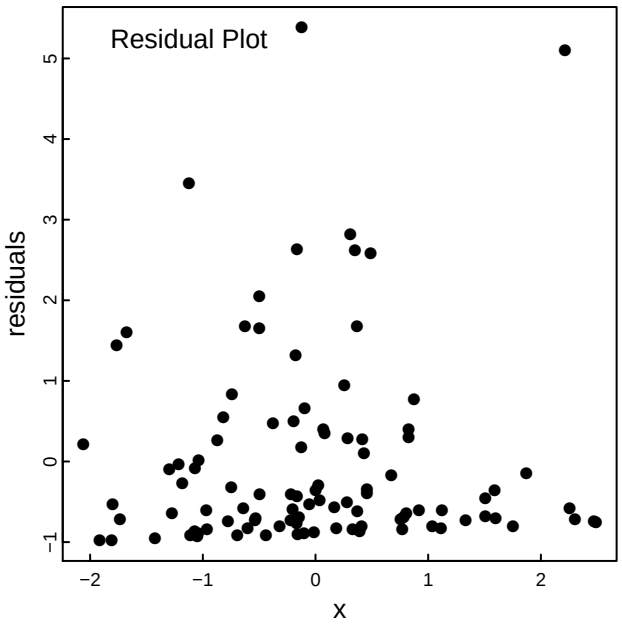

Scatter Plot
y
x
Residual Plot
residuals
x
QQ Plot
Sample Quantiles
Theoretical Quantiles

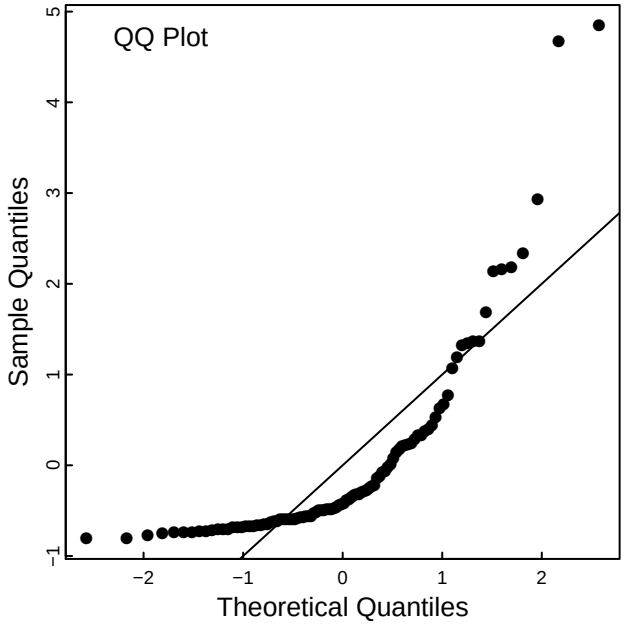

Scatter Plot
y
x
−4
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

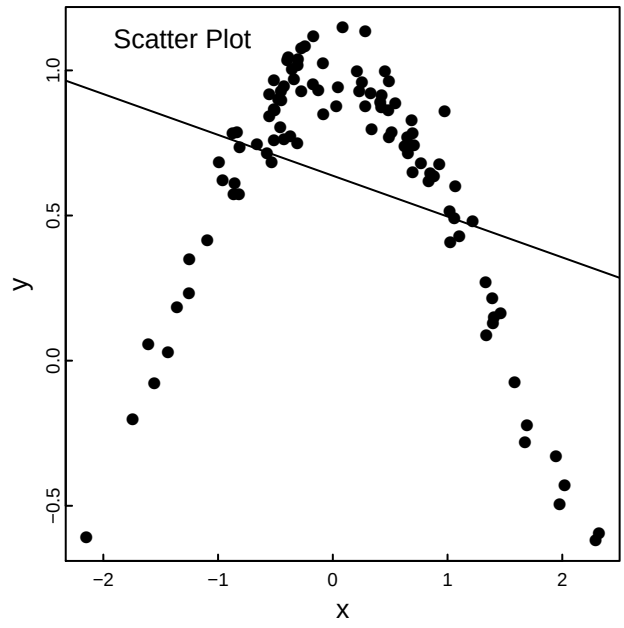

Residual Plot
residuals
x
−2
−1
0
1
2
−2
−1
0
1
2

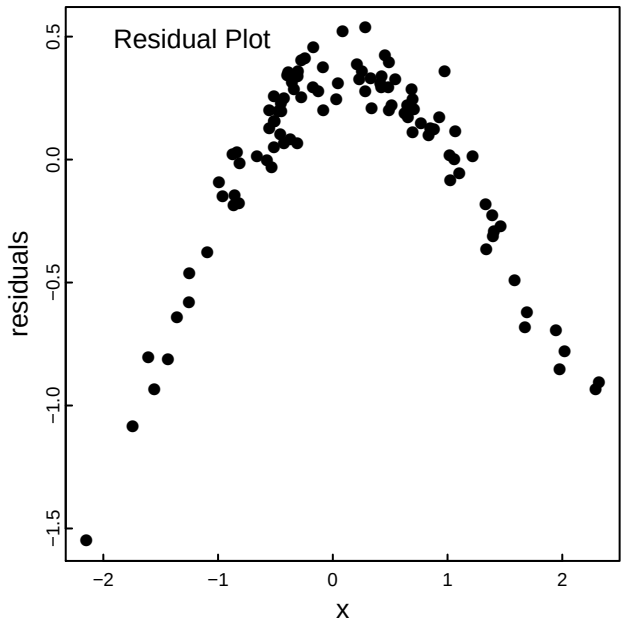

QQ Plot
Sample Quantiles
Theoretical Quantiles
−3
−2
−1
0
1
2
−2
−1
0
1
2

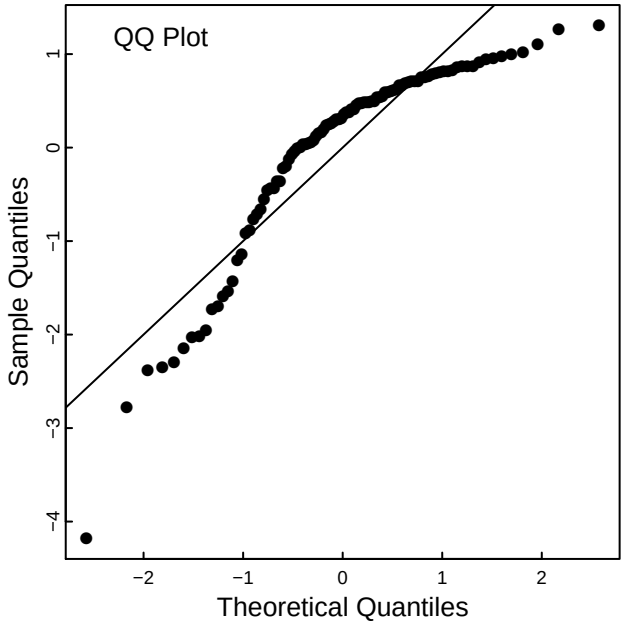

Scatter Plot
y
x
−2
−1
0
1
2
−2
0
2
4
6
8


Residual Plot
residuals
x
−2
−1
0
1
2
−1
0
1
2
3
4
5


QQ Plot
Sample Quantiles
Theoretical Quantiles
−2
−1
0
1
2
−1
0
1
2
3
4
5

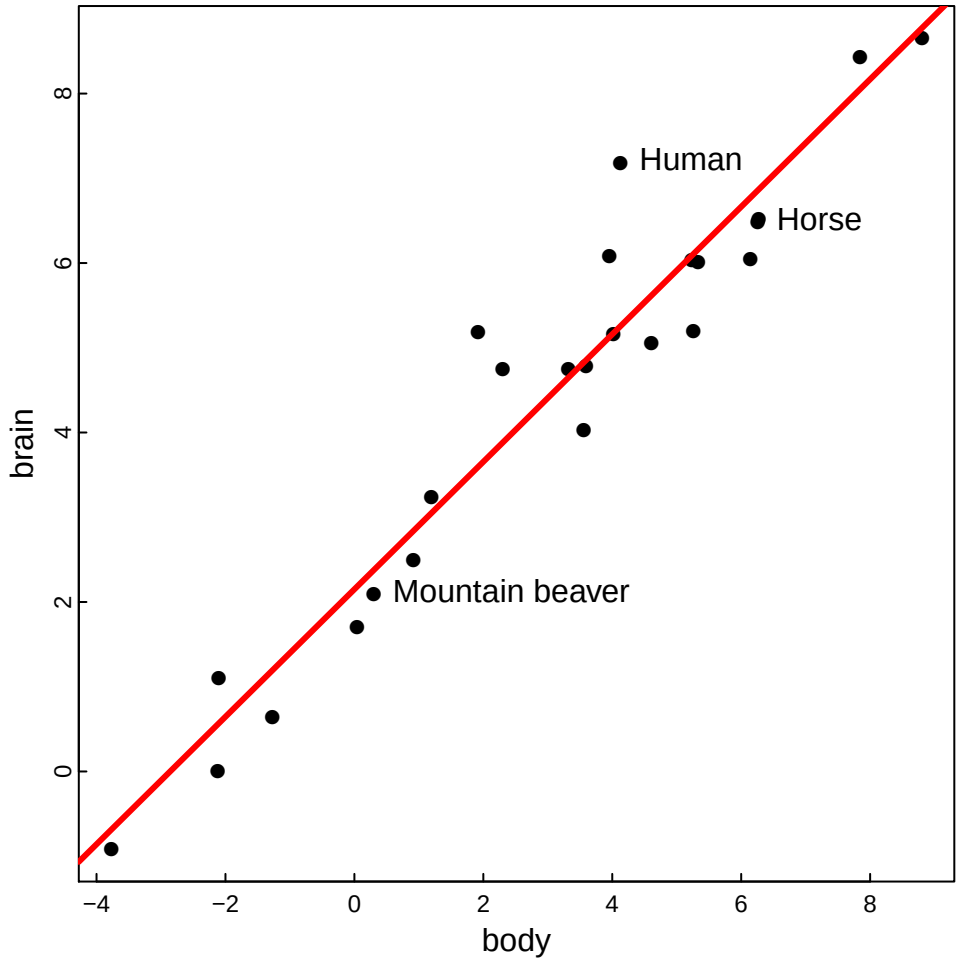

Scatter Plot
y
x
1.0
0.5
0.0
−0.5
−2
−1
0
1
2


Residual Plot
residuals
x
0.5
0.0
−0.5
−1.0
−1.5
−2
−1
0
1
2


QQ Plot
Sample Quantiles
Theoretical Quantiles
1
0
−1
−2
−3
−4
−2
−1
0
1
2

2. Consider data of log body weight (log kg) and log brain weight (log g) of 25 species of land animals. The simple linear regression results are posted below. Identify $\hat{\beta}_0$ and $\hat{\beta}_1$ and $\hat{\sigma}$. What is the interpretation of $\hat{\beta}_1$ in this context? Perform the hypothesis test $H_0 : \beta_1 = 0$ ($\alpha = 0.05$. How do you interpret this hypothesis test?

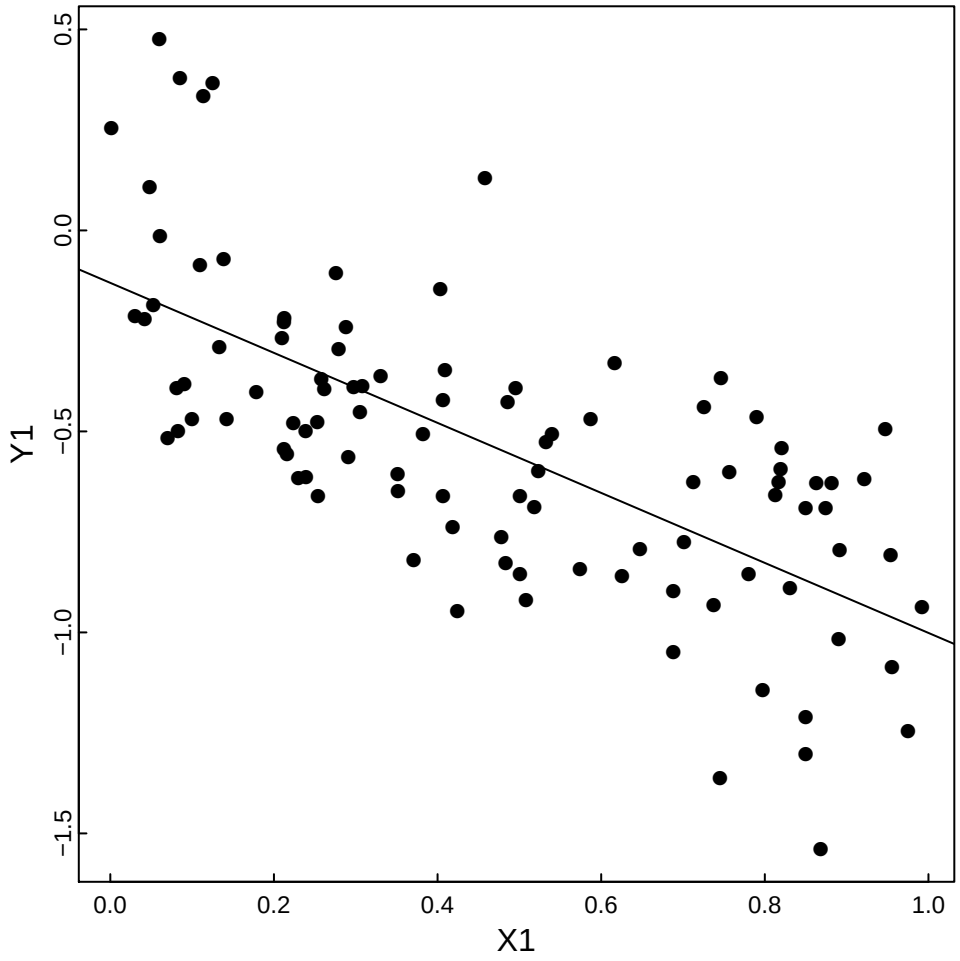
Suppose you know the log body weight of another species is 0.5. Use the linear regression model to estimate the mean log brain weight this additional animal.




```
> display(lm3)
lm(formula = log(brain) ~ log(body), data = temp)
            coef.est coef.se
(Intercept) 2.15     0.20
log(body)   0.75     0.05
---
n = 25, k = 2
residual sd = 0.73, R-Squared = 0.92

> confint(lm3)
                2.5 %    97.5 %
(Intercept) 1.7354320 2.5653922
log(body)   0.6576845 0.8468369
```

3. Consider the simple linear regression on the data presented below.



The confidence intervals of the parameter estimates are these:

```
                2.5 %      97.5 %
(Intercept) -0.2315693 -0.02943999
X1          -1.0544632 -0.68713575
```

What are the point estimates of β_0 and β_1? What is the test result of $H_0 : \beta_1 = 0$?

Solutions

1. (a) page 2
   (b) page 3
   (c) page 6
   (d) page 5
   (e) page 4

2. Identify $\hat{\beta}_0$ and $\hat{\beta}_1$ and $\hat{\sigma}$.
   $\hat{\beta}_0 = 2.15$, $\hat{\beta}_1 = 0.75$ and $\hat{\sigma} = 0.73$.

   What is the interpretation of $\hat{\beta}_1$ in this context?
   Every one unit change in log(body weight) is associated with a 0.75 change in log(brain weight).

   Perform the hypothesis test $H_0 : \beta_1 = 0$.
   Reject H_0 because 0 is not within confidence interval.

   How do you interpret this hypothesis test?
   The data suggests that log(body weight) is predictive of log(brain weight).

   Use the linear regression model to estimate the mean log brain weight this additional animal.
   $2.15 + 0.75 * 0.5 = 2.525$

3. What are the point estimates of β_0 and β_1?
   -0.13050, -0.87080

   What is the test result of $H_0 : \beta_1 = 0$?
   Reject H_0.