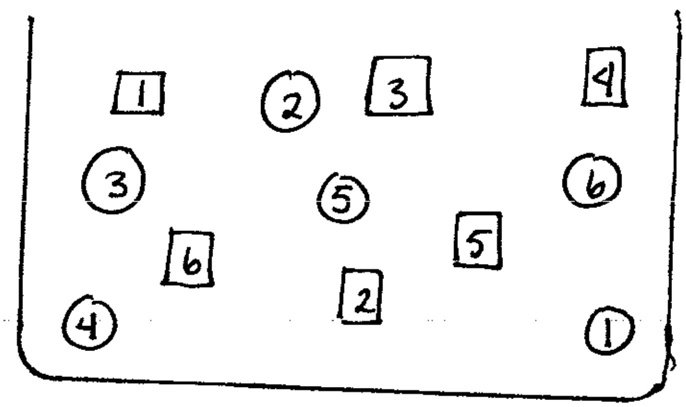
1. Consider the following population.



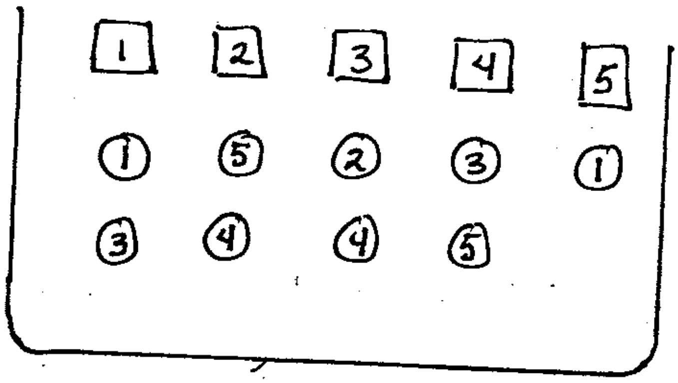
(a) Create a 2 X 6 table (cross tabulation) of shape and number.

(b) Are shape and number independent?

(c) Suppose you draw 5 units. What is the probability that the 5^{th} unit is 3 ?

(d) Suppose you draw without replacement 5 units. What is the probability that the 5^{th} unit is 3 given that the first four draws are squares?

2. Consider the following population:



(a) What must we add to this population so that shape and number are independent?

(b) Suppose we add ⑤.
Draw two units without replacement.
What is the propability that both draws are 5?

(Hint: Use multiplication rule.)

3. Consider a population where 45% are males and 55% are females. Each individual was asked:

"True or False? Groundhog Day is the greatest film ever made."

The results are reported below.

Males

True	20%
False	80%

Females

True	12%
False	88%

(a) What is the probability that a randomly selected individual responds "True"?
(Hint: Law of total probability)

(b) Is Sex and Question Response Independent?

(c) What is $P(\{\text{Male}\} \mid \{\text{True}\})$?
(Hint: Bayes Rule)

Solution on next page

Solution to 3c

Key Point: Why Bayes Rule? Because we need to find $P(\{male\} \mid \{true\})$ but the info we have is $P(\{true\} \mid \{male\})$. This is the clue to use Bayes Rule.

Bayes Rule

$$P(A \mid B) = \frac{P(A)\, P(B \mid A)}{P(B)}$$

Our Problem:

$A = \{male\} \quad B = \{True\}$

$P(A) = .45$

$P(B) =$ calculated in part (a)

$P(B \mid A) = .2$

Put it all together:

$$P(\{Male\} \mid \{True\}) = \frac{.45(.2)}{\text{ans to part (a)}}$$

HARD (Not on Quiz)

4. These are made up numbers. Suppose 20% of new mothers choose to bottlefeed their newborn without attempting to breastfeed. Suppose 30% of new mothers that do attempt to breastfeed end up choosing to bottlefeed.

(a) What is the probability that a randomly selected new mother will choose to bottlefeed?

(Hint: Law of total prob. Condition on initial choice.)

(b) (A little harder) Suppose that 10% of new mothers that initially chose to bottlefeed end up (because of a good lactation consultant) choosing to breastfeed. Find the same probability from part (a).

(Hint: same as part (a))

5. 3 coins are flipped. Calculate

P(0 heads)

P(1 heads)

P(2 heads)

P(3 heads)

6. A coin is tossed 8 times. Two possible results are

(i) H T T H T H H T

(ii) H H H H H H H H

Which is correct?

sequence (i) is more likely

sequence (ii) is more likely

the sequences are equally likely

6. Two dice are rolled.

Let $\{A\} = \{ 1^{st} \text{ die} = 1 \}$

$\{B\} = \{ 2^{nd} \text{ die} = 2 \}$

What is $P(\{A\} \cup \{B\})$?

$P(\{A\} \cap \{B\})$?

7. Completely Made up

Mother's Race	Newborn Weight (g) [0, 1500)	[1500, 3000)	[3000, Inf)
Black	29	101	35
White	41	360	82
Asian	8	92	11
Other	11	71	16

I don't know anything about newborns or newborn's mothers. However, suppose weight

less than 1500 g
or
greater than or equal to 3000 g

is considered extreme.

Calculate

(a) P(Black | < 1500 g)

(b) P([1500, 3000))

(c) P([1500, 3000) | White)

(d) P(≥ 1500 g | Black or White)

(e) P(extreme | Not Asian)

(f) P(extreme | Asian)

(g) P(Asian)

(h) P(extreme)

(i) Calculate P(Asian | extreme) using Bayes part ~~[illegible]~~ (f), (g), (h).

Note: In class we said

$$P(\{A\}) = \text{proportion of population for which "}\{A\}\text{ is true"}$$

Let $\overline{\{A\}}$ denote those outcomes where "$\{A\}$ is false"

~~One trick in probability is to calculate~~

This means

$$P(\{A\}) = 1 - P(\overline{\{A\}})$$

One trick is to calculate $P(\overline{\{A\}})$ to find $P(\{A\})$.

Try it:

Draw 2 cards — with replacement — from standard deck.

Find $P(\text{at least 1 heart})$.