Practice problems from HBSP

work one or two problems from each box.

10.10
10.14
10.18
10.21
10.24
10.27
10.30

10.58
10.61
10.64
10.67
10.70

11.19
11.21
11.23
11.25
11.27
11.29

10.32
10.36
10.38
10.40
10.45
10.48
10.51
10.54

11.32
11.36

11.49
11.52
11.55
11.58

1. A food label indicates Burger King's Whopper has 670 calories. A researcher claims that the average Whopper has more. He measures the calories for 20 randomply sampled Whoppers and finds the average calorie count to be 679. The sample SD is 57.

2. An educator claims that more than 5% of third graders are not reading at grade level. To test the claim, the school district randomly selects 100 students and finds that 2 are not proficient readers.

3. A coin is flipped 1000 times and lands heads 589 times. A student wants to test whethter the coin is fair.

4. Suppose olive oil is classified as pure, virgin, or extra virgin based on a *purity* score. A chef is concerned that a brand of extra virgin olive oil is not quite up to snuff. He thinks the average purity measure is lower than the score of 95 it needs to be classified as *extra virgin*. In order to explore this hypothesis, the chef randomly samples 15 different bottles (from different lots) and performs an olive oil purity test. He finds the average purity score of his samples to be 91. The SD is 2.

5. Suppose the government claims that house prices have remained steady in the last year. To test this claim, an economist randomly selects 100 homes and ascertains how the value of each home has changed in the past year. (In some cases it was negative; in others it was positive.) The average change in home price from his data was \$10 K. The SD was [illegible] K.

20

6. A researcher claims the average elementary student only consumes 300 calories for breakfast on a school day. To investigate this claim, another researcher determines the caloric intake of 50 elementary school children on a school day. She calculates a sample average of 407 and a sample SD of 216.

7. A random sample of 50 North Carolina voters where asked: "If the election where today, would you vote for President Obama?" 22 of the voters said yes. Does this data support the claim that President Obama ~~will win the North Carolina vote?~~ has more than 50% support from NC voters?

(0) What is the significance level?

$\alpha = .05$

1. (a) What is the hypothesis?

$$H_0: \mu \leq 670$$
$$H_1: \mu > 670$$

(b) Which test statistic?

$$T = \sqrt{n}\,\frac{\bar{X} - 670}{S}$$

n=20 is smallish, but assuming X~Normal is probably o.k. for this data.

$$= \sqrt{20}\,\frac{679-670}{57}$$

$$= 0.71$$

(c) What does "extreme" mean?

Values to the right.

(d) Calculate p-value.

$$\text{p-value} = P(T \geq 0.71)$$

where $T \sim t(19)$

From Table B5 (in back of book)

$$P(T \geq 1.729) = .05$$

So p-value $= P(T \geq 0.71) > P(T \geq 1.729) = .05$

(e) Conclustion.

p-value > 0.05

"we fail to reject"

$\boxed{2}$ (a) What is the significance level?

$\alpha = .01$

(b) What is the hypothesis?

$H_0 : p \leq 0.05$

$H_1 : p > 0.05$

(c) Which test statistic?

$$Z = \frac{\sqrt{n}\,(Y - n p_0)}{\sqrt{Y\,(n-Y)}}$$

$$Z = \frac{\sqrt{100}\,(2-5)}{\sqrt{2 * 98}}$$

$$= -2.14$$

(d) What does "extreme" mean?

values to the right

(e) Calculate p-value.

$p\text{-value} = P(Z \geq -2.14) = 0.9838$

from Z-table

(f) Conclusion

"Fail to reject H_0"

3

(a) What is the significance level?

$\alpha = 0.01$

(b) What are the hypotheses?

$H_0: p = .5$

$H_1: p \neq .5$

(c) Which test statistic?

$$Z = \sqrt{n}\,\frac{(Y - np_0)}{\sqrt{Y(n-Y)}}$$

$$= \sqrt{1000}\,\frac{(589 - 500)}{\sqrt{589 \times 411}}$$

$$= 5.72$$

(d) What does "extreme" mean?

Tails

(e) Calculate p-value

$$\text{p-value} = P(Z \geq 5.72) + P(Z < -5.72)$$

= very, very small.

< 0.0001

(f) Conclusion

"Reject H_0"

$\boxed{4}$

(a) Significance Level

$$\alpha = 0.07$$

(b) Hypotheses

$$H_0: \mu \geq 95$$
$$H_1: \mu < 95$$

(c) Test Statistic

$$T = \frac{\sqrt{n}\,(\bar{X} - \mu_0)}{S}$$

$$= \frac{\sqrt{15}\,(91-95)}{2}$$

$$= -7.7$$

(d) What does "extreme" mean?

Values to the left.

(e) Calculate p-value.

p-value $= P(T \leq -7.7)$ where $T \sim t(14)$

$=$ very, very small

< 0.0001

(f) Conclusion

"Reject H_0"

$\boxed{5}$

(a) significance level

$$\alpha = 0.02$$

(b) Hypotheses

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

(c) Test Statistic

$$T = \sqrt{n}\ \frac{\bar{X} - \mu_0}{S}$$

$$= \sqrt{100}\ \frac{10 - 0}{20}$$

$$= 5$$

(d) What does "extreme" mean?

Tails

(e) Calculate p-value.

$$\text{p-value} = P(T \geq 5) + P(T \leq -5)$$
$$= \text{very small}$$
$$< 0.0001$$

(f) Conclusion

"Reject H_0"

$\boxed{6}$ (a) Significance Level

$\alpha = 0.01$

(b) Hypotheses

$H_0 : \mu = 300$

$H_1 : \mu \neq 300$

(c) Test Statistic

$$TS = \frac{\sqrt{50}\,(407 - 300)}{216}$$

$$= 3.5$$

(d) What is "extreme"?

Tails

(e) Calculate p-value.

$$\text{p-value} = P(T \geq 3.5) + P(T \leq -3.5)$$

Where $T \sim t(49)$

$$= 0.0010$$

(f) Conclusion

"Reject H_0"

$\boxed{7}$ (a) Significance Level

$\alpha = 0.01$

(b) Hypotheses

$H_0: \varphi \leq .5$

$H_1: \varphi > .5$

(c) Test Statistic

$$Z = \frac{\sqrt{50}\,(22 - 25)}{\sqrt{22 * 28}}$$

$$= -.85$$

(d) More Extreme ?

Right side

(e) P-value

$$\text{p-value} = P(Z \geq -.85)$$
$$= 0.8023$$

(f) Conclusion

"Fail to Reject H_0"

1. Researchers collect income figures from a random sample of 200 men and a random sample of 200 females. The researchers are interested in income equality, particularly in the hypothesis that women earn less then men. They calculate $\bar{x}_m$ = \$58 K and $\bar{x}_f$ = \$53 K. Futher, s_m = 12 K and s_f = 20 K.

2. An instructor randomly and evenly distributes two versions of a test to a class of ~~120~~ 240 students. He is interested to see if the tests differ in difficulty.

   $\bar{X}_1 = 69.5$ $\quad\bar{X}_2 = 70.9$

   $S_1^2 = 116$ $\quad S_2^2 = 370$

3. An instructor randomly and evenly distributes two versions of a test to a class of ~~120~~ 240 students. He is interested to see if the number of As differ between the two tests.

   Test 1 : # As = 25

   Test 2 : # As = 20

1

(a) Significance Level

$\alpha = 0.06$

(b) Hypotheses

$H_0: \mu_m - \mu_f \leq 0$

$H_1: \mu_m - \mu_f > 0$

(c) Test Statistic

$$T = \sqrt{n_1 n_2}\ \frac{(\bar{X}_m - \bar{X}_f) - 0}{\sqrt{n_2 s_1^2 + n_1 s_2^2}}$$

$$= 200\ \frac{5}{\sqrt{200(12^2 + 20^2)}}$$

$$= 3.03$$

(d) Extreme ?

To the right

(e) p-value

can use normal approx because n is large.

$$\text{p-value} = P(Z \geq 3.03)$$

$$= 0.0012$$

(f) conclusion

"Reject H_0"

$\boxed{2}$

(a) Significance Level

$\alpha = 0.05$

(b) Hypotheses

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

(c) Test Statistic

$$T = \sqrt{n_1 n_2}\ \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{n_2 s_1^2 + n_1 s_2^2}}$$

$$= 120 \left(\frac{-1.4}{\sqrt{120(116+370)}} \right)$$

$$= -0.70$$

(d) Extreme?

Tails

(e) p-value

Large $n \Rightarrow$ Normal Approx

$$\text{p-value} = P(Z \leq -0.70) + P(Z \geq 0.7)$$

$$= 2 * P(Z \leq -0.7)$$

$$= 0.4839$$

(f) Conclusion

"Fail to Reject H_0"

$\boxed{3}$ (a) Significance Level

$\alpha = 0.05$

(b) Hypotheses

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

(c) Test Statistic

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - p_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

$$= \frac{\frac{25-20}{120}}{\sqrt{\frac{25 * 95}{120^3} + \frac{20 * 100}{120^3}}}$$

$$= \frac{5}{\sqrt{\frac{25 * 95 + 20 * 100}{120}}}$$

$= 0.83$

(d) Extreme?

Tails

(e) p-value

p-value $= P(Z \geq 0.83) + P(Z \leq -0.83)$

$= 0.41$

(f) Conclusion "Fail to reject H_0"