

## CHAPTER 6

### Exercise 7

- a.  $A \cap B$  is the event that the individual is exposed to high levels of both carbon monoxide and nitrogen dioxide.
- b.  $A \cup B$  is the event that the individual is exposed to either carbon monoxide or nitrogen dioxide or both.
- c.  $A^c$  is the event that the individual is not exposed to high levels of carbon monoxide.
- d. The events  $A$  and  $B$  are not mutually exclusive.

### Exercise 9

- a. The probability that a woman who gave birth in 1992 was 24 years of age or younger is

$$\begin{aligned}P(\leq 24) &= P(< 15 \text{ or } 15-19 \text{ or } 20-24) \\&= P(< 15) + P(15-19) + P(20-24) \\&= 0.003 + 0.124 + 0.263 \\&= 0.390.\end{aligned}$$

- b. The probability that the woman was 40 years of age or older is

$$\begin{aligned}P(\geq 40) &= P(40-44 \text{ or } 45-49) \\&= P(40-44) + P(45-49) \\&= 0.014 + 0.001 \\&= 0.015.\end{aligned}$$

- c. Given that the woman was under 30 years of age, the probability that she was not yet 20 is

$$\begin{aligned}P(< 20 | < 30) &= \frac{P(< 20 \text{ and } < 30)}{P(< 30)} \\&= \frac{P(< 20)}{P(< 30)} \\&= \frac{0.003 + 0.124}{0.003 + 0.124 + 0.263 + 0.290} \\&= 0.187.\end{aligned}$$

- d. Given that the woman was 35 years of age or older, the probability that she was under 40 is

$$\begin{aligned}P(< 40 | \geq 35) &= \frac{P(< 40 \text{ and } \geq 35)}{P(\geq 35)} \\&= \frac{P(35-39)}{P(\geq 35)} \\&= \frac{0.085}{0.085 + 0.014 + 0.001} \\&= 0.850.\end{aligned}$$

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### Exercise 11

- Since these events are independent, the probability that both adults are uninsured is

$$\begin{aligned} P(\text{both uninsured}) &= P(\text{woman uninsured}) \times P(\text{man uninsured}) \\ &= 0.123 \times 0.123 \\ &= 0.015. \end{aligned}$$

- The probability that both adults are insured is

$$\begin{aligned} P(\text{both insured}) &= (1 - 0.123) \times (1 - 0.123) \\ &= 0.877 \times 0.877 \\ &= 0.769. \end{aligned}$$

- The probability that all five adults are uninsured is  $0.123 \times 0.123 \times 0.123 \times 0.123 \times 0.123 = 0.000028$ .

### Exercise 13

- The probability of a false negative result is

$$\begin{aligned} P(- \text{ test} | \text{ ca}) &= 1 - \text{sensitivity} \\ &= 1 - 0.85 \\ &= 0.15. \end{aligned}$$

- The probability of a false positive result is

$$\begin{aligned} P(+ \text{ test} | \text{ no ca}) &= 1 - \text{specificity} \\ &= 1 - 0.80 \\ &= 0.20. \end{aligned}$$

- Since  $P(\text{ca}) = 0.0025$  and  $P(\text{no ca}) = 0.9975$ , the probability that a woman has breast cancer given that her mammogram is positive is

$$\begin{aligned} P(\text{ca} | + \text{ test}) &= \frac{P(\text{ca})P(+ \text{ test} | \text{ ca})}{P(\text{ca})P(+ \text{ test} | \text{ ca}) + P(\text{no ca})P(+ \text{ test} | \text{ no ca})} \\ &= \frac{(0.0025)(0.85)}{(0.0025)(0.85) + (0.9975)(0.20)} \\ &= 0.0105. \end{aligned}$$

### Exercise 15

- The sensitivity of radionuclide ventriculography is

$$\begin{aligned} P(+ \text{ rv} | \text{ cad}) &= \frac{302}{481} \\ &= 0.628, \end{aligned}$$

and its specificity is

$$\begin{aligned} P(- \text{ rv} | \text{ no cad}) &= \frac{372}{452} \\ &= 0.823. \end{aligned}$$

b. Since  $P(\text{cad}) = 0.10$  and  $P(\text{no cad}) = 0.90$ , the probability that an individual has coronary artery disease given that he or she tests positive is

$$\begin{aligned} P(\text{cad} \mid +rv) &= \frac{P(+rv \mid \text{cad})P(\text{cad})}{P(+rv \mid \text{cad})P(\text{cad}) + P(+rv \mid \text{no cad})P(\text{no cad})} \\ &= \frac{(0.628)(0.10)}{(0.628)(0.10) + (0.177)(0.90)} \\ &= 0.283. \end{aligned}$$

c. The predictive value of a negative test is

$$\begin{aligned} P(\text{no cad} \mid -rv) &= \frac{P(-rv \mid \text{no cad})P(\text{no cad})}{P(-rv \mid \text{no cad})P(\text{no cad}) + P(-rv \mid \text{cad})P(\text{cad})} \\ &= \frac{(0.823)(0.90)}{(0.823)(0.90) + (0.372)(0.10)} \\ &= 0.952. \end{aligned}$$

### Exercise 17

- a. In Brooklyn, the probability of a positive test result is 0.0129, or 1.29%.  
b. The prevalence of HIV infection is

$$\begin{aligned} P(H) &= \frac{P(+ \text{ test}) - [1 - \text{specificity}]}{\text{sensitivity} - [1 - \text{specificity}]} \\ &= \frac{0.0129 - [1 - 0.998]}{0.99 - [1 - 0.998]} \\ &= 0.011. \end{aligned}$$

### Exercise 19

- a. The probabilities of suffering from persistent respiratory symptoms by socioeconomic status are shown below.

Socioeconomic Status	Probability
Low	0.392
Middle	0.238
High	0.141

- b. Let  $S$  represent the presence of symptoms. The odds of experiencing persistent respiratory symptoms for the middle group relative to the high group are

$$\begin{aligned} \text{OR} &= \frac{P(S \mid \text{middle})/[1 - P(S \mid \text{middle})]}{P(S \mid \text{high})/[1 - P(S \mid \text{high})]} \\ &= \frac{(0.238)/(1 - 0.238)}{(0.141)/(1 - 0.141)} \\ &= 1.90, \end{aligned}$$

b. Since  $P(\text{cad}) = 0.10$  and  $P(\text{no cad}) = 0.90$ , the probability that an individual has coronary artery disease given that he or she tests positive is

$$\begin{aligned} P(\text{cad} | +rv) &= \frac{P(+rv | \text{cad})P(\text{cad})}{P(+rv | \text{cad})P(\text{cad}) + P(+rv | \text{no cad})P(\text{no cad})} \\ &= \frac{(0.628)(0.10)}{(0.628)(0.10) + (0.177)(0.90)} \\ &= 0.283. \end{aligned}$$

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and for the low group relative to the high group are

$$\begin{aligned} \text{OR} &= \frac{P(S | \text{low})/[1 - P(S | \text{low})]}{P(S | \text{high})/[1 - P(S | \text{high})]} \\ &= \frac{(0.392)/(1 - 0.392)}{(0.141)/(1 - 0.141)} \\ &= 3.93. \end{aligned}$$

c. There does appear to be an association between socioeconomic status and respiratory symptoms; the odds of experiencing symptoms increase as socioeconomic status decreases.