

1. (5.8) (p. 97)

- (a) Let
- X
- be the age of mothers of newborns in the United States. Then,

$$\Pr(X \leq 19) = \Pr(X \leq 14) + \Pr(14 < X \leq 19) = 0.003 + 0.1 = 0.103$$

Thus, if we select a birth at random, the probability that the mother is less than or equal to 19 years of age is 0.103.

- (b)
- $\Pr(X \geq 30) = 0.3 + 0.097 = 0.397$

If we select a birth at random, the probability that the mother is at least 30 years of age is 0.397.

- (c) The expected age of mothers of newborns in the U.S. is 28.91 years.

$$\begin{aligned} \mu &= \sum_x x \cdot \Pr(X = x) \\ &= 12.5(0.003) + 17.5(0.1) + 25(0.5) + 35(0.3) + 42.5(0.097) + 47.5(0.000) \\ &= 0.0375 + 1.75 + 12.5 + 10.5 + 4.1225 + 0 \\ &= 28.91 \text{ years} \end{aligned}$$

- (d) The variance in age of mothers of newborns in the U.S. is 50.5119 years
- ²
- .

$$\begin{aligned} \sigma^2 &= \sum_x (x - \mu)^2 \cdot \Pr(X = x) \\ &= (12.5 - 28.91)^2(0.003) + (17.5 - 28.91)^2(0.1) + (25 - 28.91)^2(0.5) + \\ &\quad + (35 - 28.91)^2(0.3) + (42.5 - 28.91)^2(0.097) + (47.5 - 28.91)^2(0.000) \\ &= 0.8079 + 13.0188 + 7.6441 + 11.1264 + 17.9147 + 0 \\ &= 50.5119 \text{ years}^2 \end{aligned}$$

2. (5.10) (p. 99)

$$\Pr(\text{individual self-identifies as Hispanic}) = 0.000 + 0.003 + 0.060 + 0.062 = 0.125$$

3. (5.12) (p. 103) - Refer to Figure 5.7

$$(a) \Pr(X \geq 0.6) = \text{base} \times \text{height} = (1 - 0.6)(1) = 0.4$$

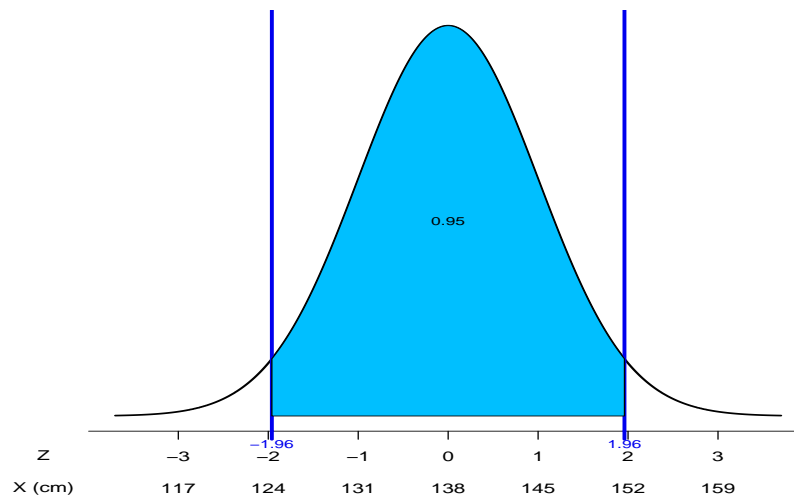
$$(b) \Pr(X \leq 0.6) = \text{base} \times \text{height} = (0.6)(1) = 0.6$$

4. (7.2) (p. 139) Using the 68–95–99.7% approximation rule, we calculate the following.

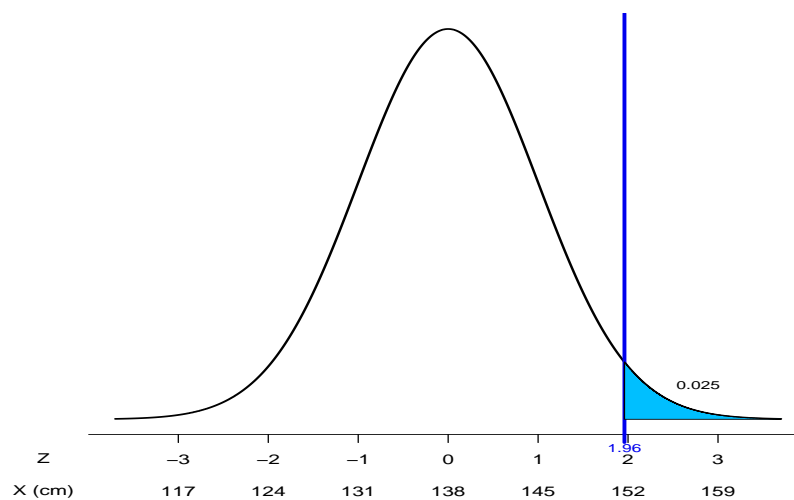
- 1) The middle 95% of heights are captured by the range
 $\mu \pm 2(\sigma) = 138 \pm 2(7) = (124, 152)$ cm.
- 2) The tallest 2.5% are taller than $\mu + 2(\sigma) = \mu + 2(7) = 152$ cm.

Alternatively, we can base the calculations on Table B (exact).

- 1) The middle 95% of heights are captured by the range
 $\mu \pm 2(\sigma) = 138 \pm 1.96(7) = (124.28, 151.72)$ cm.

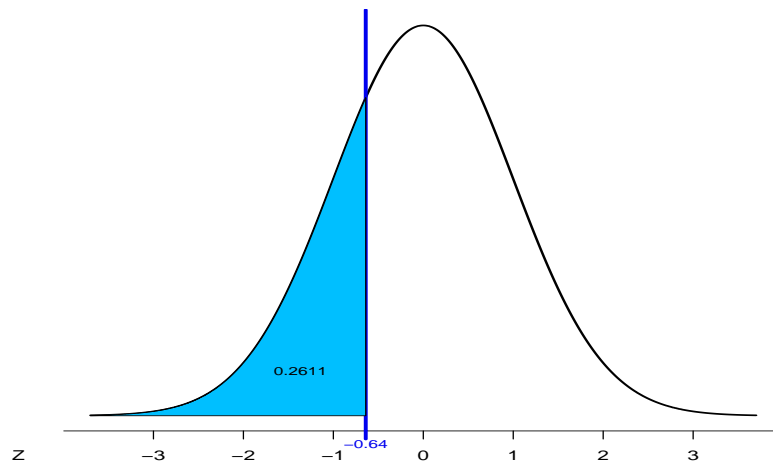


- 2) The tallest 2.5% are taller than $\mu + 1.96(\sigma) = \mu + 1.96(7) = 151.72$ cm.

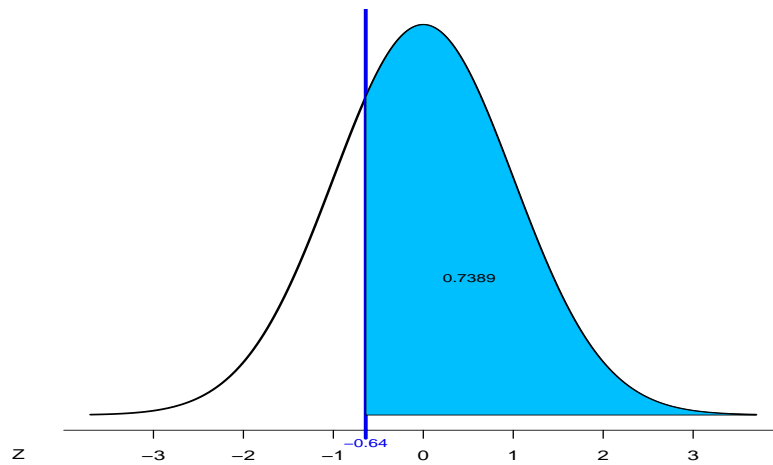


5. (7.4) (p. 143)

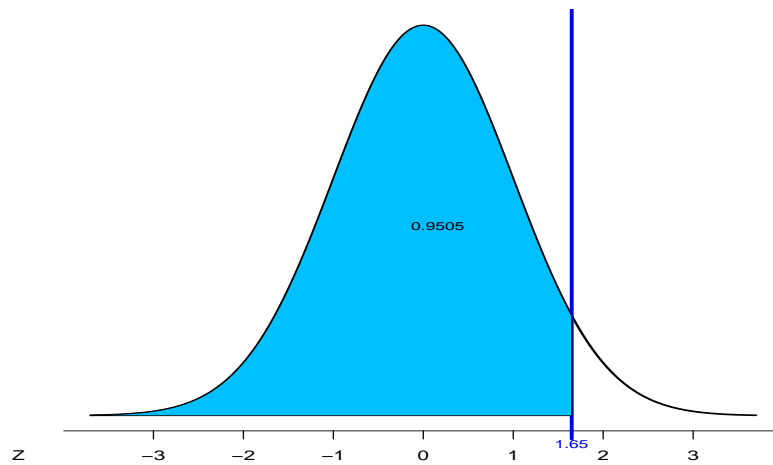
(a) $\Pr(Z < -0.64) = 0.2611$



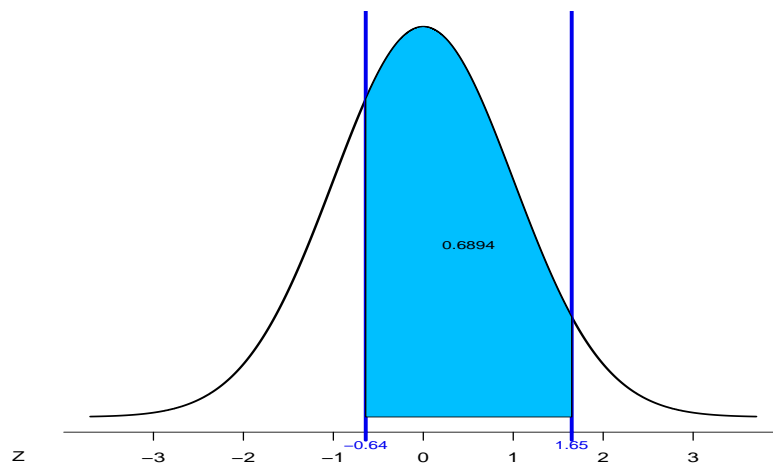
(b) $\Pr(Z > -0.64) = 1 - 0.2661 = 0.7339$



(c) $\Pr(Z < 1.65) = 0.9505$



(d) $\Pr(-0.64 < Z < 1.65) = \Pr(Z < 1.65) - \Pr(Z < -0.64) = 0.9505 - 0.2611 = 0.6894$



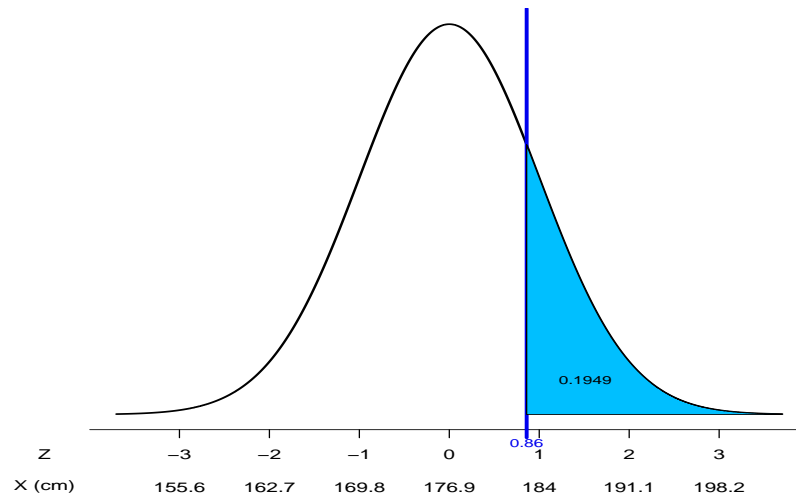
6. (7.6) (p. 145) - Sketch curves for parts a,b

(a) Let X represent the heights of 20 year old males such that $X \sim N(176.9, 7.1)$.

The quantity of interest is $\Pr(X > 183)$.

The standardized quantity is $\Pr(Z > \frac{183-176.9}{7.1}) = \Pr(Z > 0.86)$.

The sketch of this probability is



Thus from Table B, $\Pr(Z > 0.86) = 0.1949$ (about 19.49%).

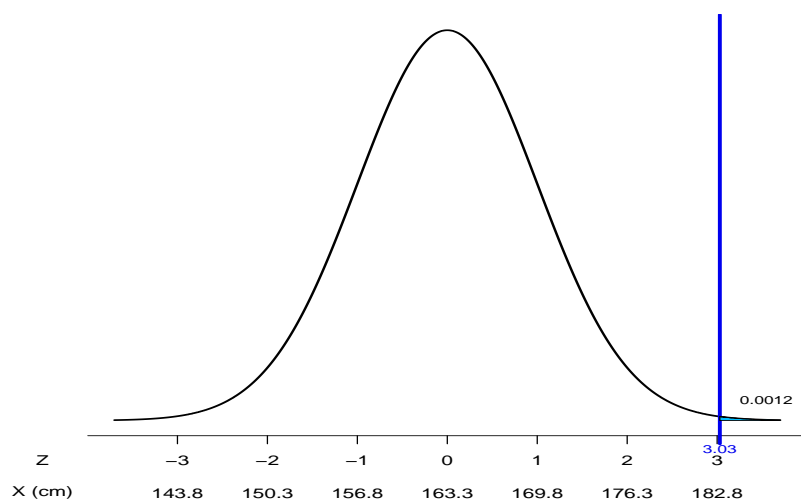
Therefore, 19.49% of U.S. men are at least 6 ft tall by age 20.

(b) Let Y be the height of 20 year old women such that $Y \sim N(163.3, 6.5)$.

The quantity of interest is $\Pr(Y > 183)$.

The standardized quantity is $\Pr(Z > \frac{183-163.3}{6.5}) = \Pr(Z > 3.03)$.

The sketch of the probability is



From Table B, $\Pr(Z > 3.03) = 0.0012$.

Therefore, only 0.12% of U.S. women are taller than 6 ft at age 20.

- (c) One out of every 5 20-year old U.S. men are at least 6 ft tall, while only 1 out of every 1000 20-year-old U.S. women are at least 6 ft tall.

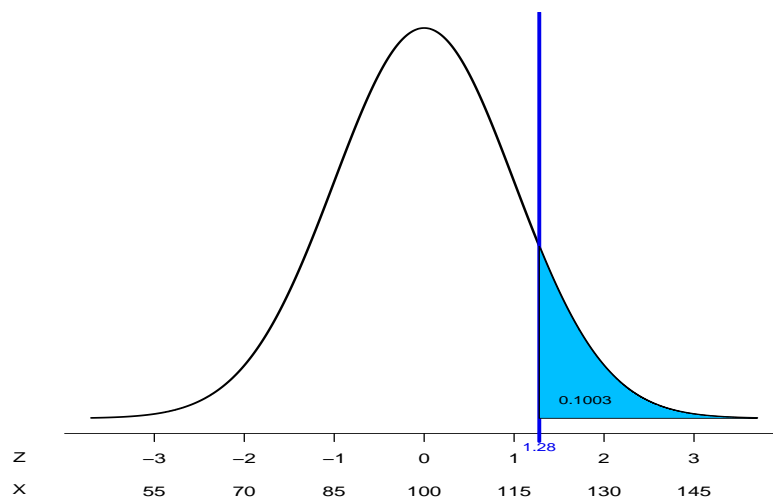
7. (7.10) (p. 147) - Sketch curves for both questions

Let X be the Wechsler Adult Intelligence Score such that $X \sim N(100, 15)$.

- 1) The quantity of interest is a such that $\Pr(X > a) = 0.1$. Equivalently, we want a such that $\Pr(X \leq a) = 0.9$.

The standardized quantity is a such that $\Pr(Z \leq \frac{a-100}{15}) = 0.9$.

The sketch of the probability is



From Table B, $z_{0.9} = 1.28$.

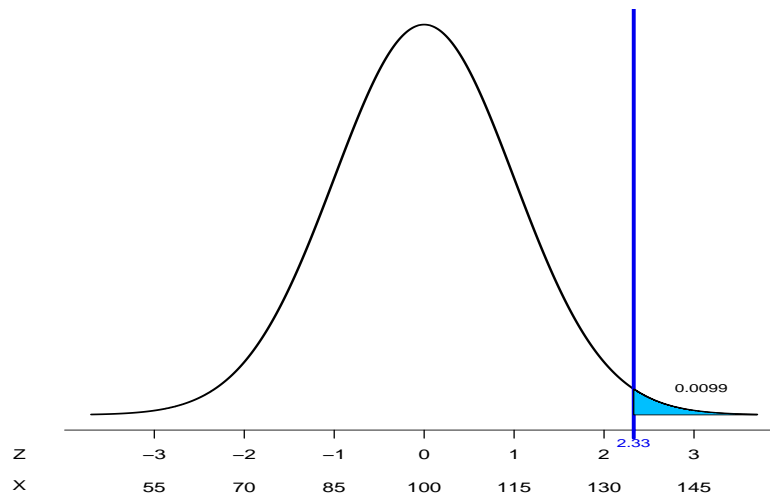
Unstandardization yields $a = \mu + z_{0.9}\sigma = 100 + 1.28(15) = 119.2$.

Therefore, scores above 119.2, or 120, are in the top 10% of scores.

- 2) The quantity of interest is a such that $\Pr(X > a) = 0.01$. Equivalently, we want a such that $\Pr(X \leq a) = 0.99$.

The standardized quantity is a such that $\Pr(Z \leq \frac{a-100}{15}) = 0.99$.

The sketch of the probability is



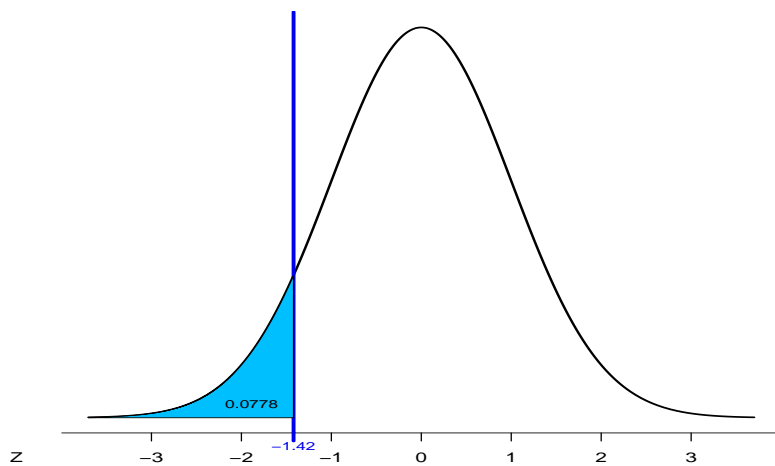
From Table B, $z_{0.99} = 2.33$.

Unstandardization yields $a = \mu + z_{0.99}\sigma = 100 + 2.33(15) = 134.95$.

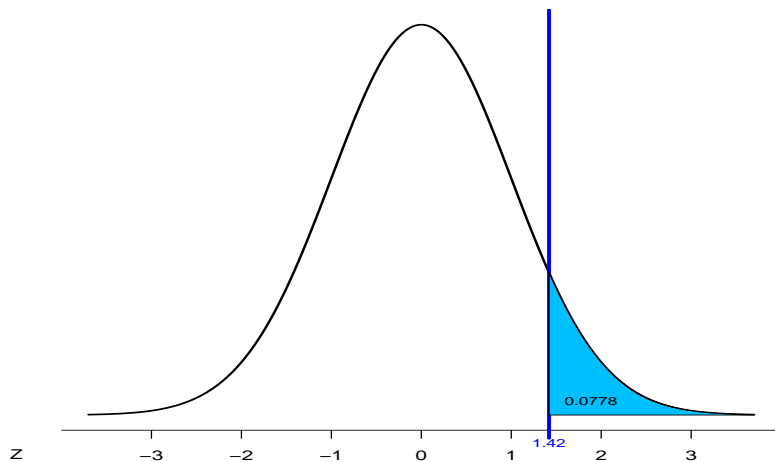
Therefore, scores above 134.95, or 135, are in the top 1% of scores.

8. (7.12) (p. 153) - Sketch curves

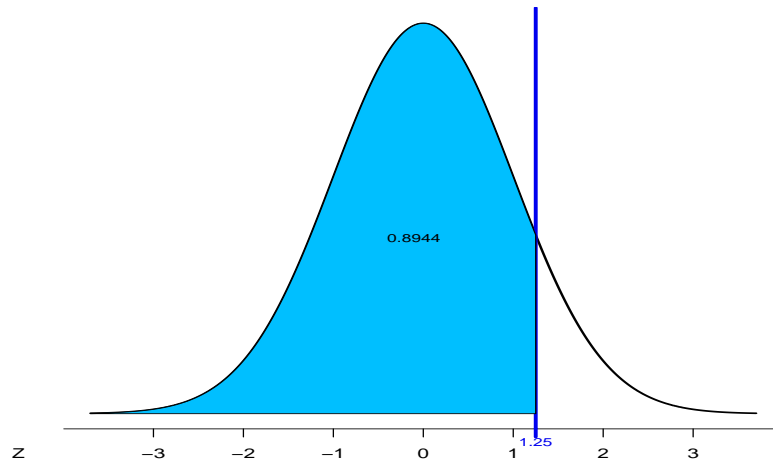
(a) $\Pr(Z < -1.42) = 0.0778$



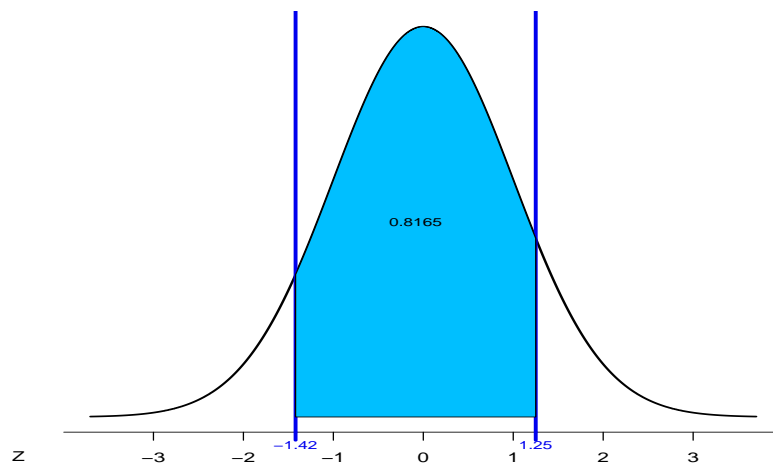
(b) $\Pr(Z > 1.42) = 1 - \Pr(Z \leq 1.42) = 1 - 0.9222 = 0.0778$



(c) $\Pr(Z < 1.25) = 0.8944$



(d) $\Pr(-1.42 < Z < 1.25) = \Pr(Z < 1.25) - \Pr(Z < -1.42) = 0.8944 - 0.0778 = 0.8166$



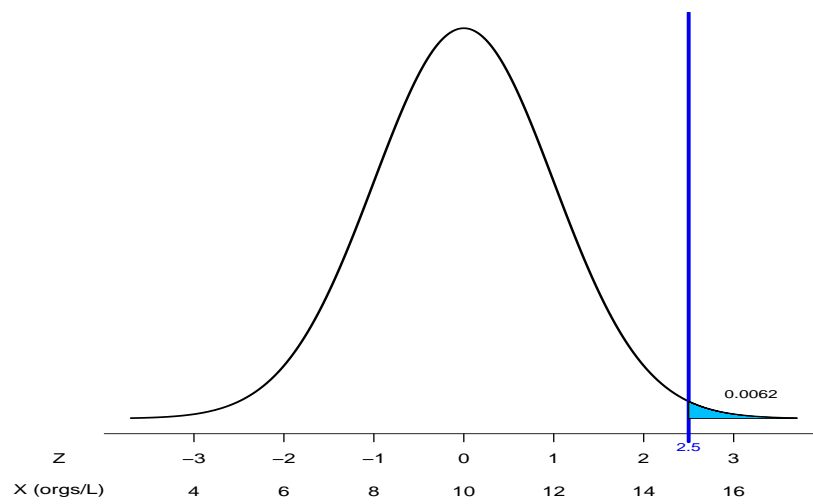
9. (7.14) (p. 153) - Sketch curve

Let X be the coliform level in a site such that $X \sim N(10, 2)$.

The quantity of interest is $\Pr(X > 15)$.

The standardized quantity is $\Pr(Z > \frac{15-10}{\sqrt{2}}) = \Pr(Z > 2.5)$.

The sketch of the probability is



From Table B, $\Pr(Z > 2.5) = 1 - 0.9938 = 0.0062$.

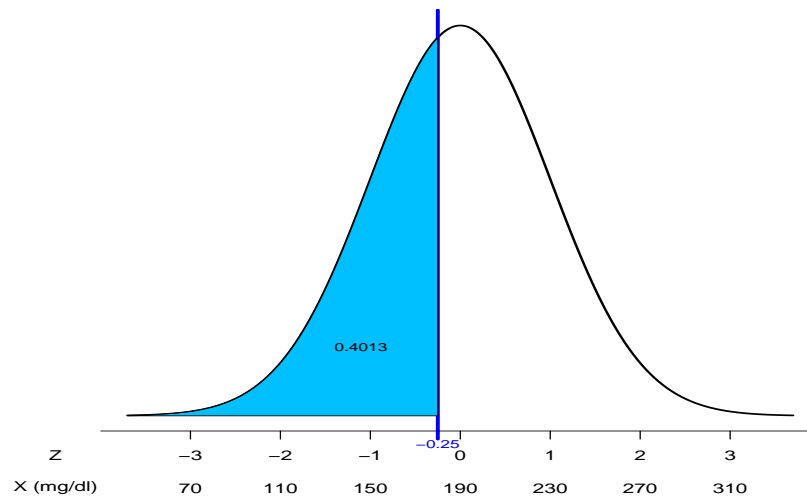
Therefore, there are 0.62% of samples with more than 15 organisms.

10. (8.6) (p. 166) Let X represent the serum cholesterol (mg/dl) in undergraduate men, such that $X \sim N(190, 40)$.

(a) The quantity of interest is $\Pr(X \leq 180)$

The standardized quantity is $\Pr(Z \leq -0.25)$

The sketch of the probability is



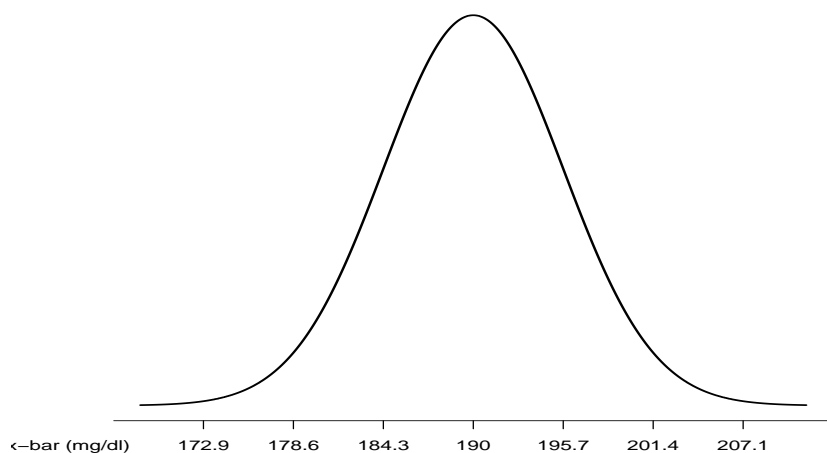
From Table B, $\Pr(Z \leq -0.25) = 0.4013$

Therefore, the probability of randomly selecting an undergraduate man with serum cholesterol less than 180 is 0.4013.

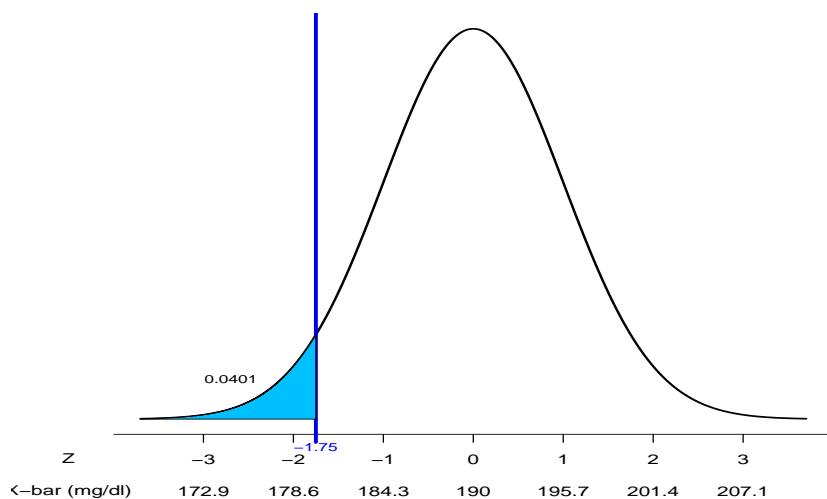
- (b) To obtain the sampling distribution of \bar{x} , first calculate the standard deviation of \bar{x} .

$$\sigma_{\bar{X}} = \frac{40}{\sqrt{49}} = \frac{40}{7} = 5.7143$$

By Chapter 8 results, we know that $\bar{X} \sim N\left(190, \frac{40}{7}\right)$.



- (c) The quantity of interest is $\Pr(\bar{X} \leq 180)$
 The standardized quantity is $\Pr(Z \leq -1.75)$
 The sketch of the probability is



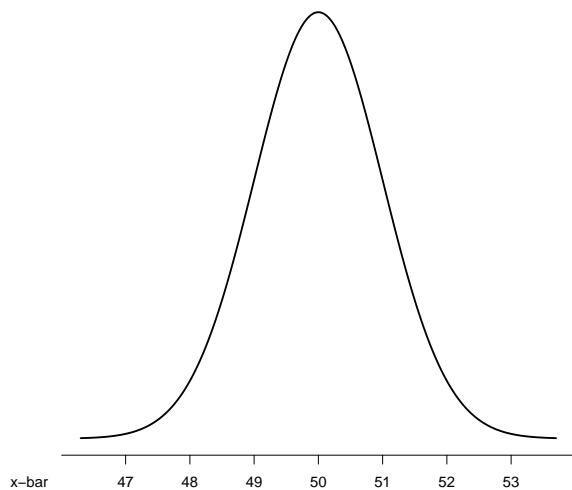
From Table B, $\Pr(Z \leq -1.75) = 0.0401$

Therefore, the probability of getting a sample mean less than 180 is 0.0401.

11. (8.8) (p. 167)

- (a) $\bar{X} \sim N\left(50, \frac{5}{\sqrt{25}}\right) \equiv N\left(50, \frac{5}{5}\right) \equiv N(50, 1)$. That is, the sample mean \bar{X} is distributed Normally with mean $\mu = 50$ and standard deviation $SE_{\bar{X}} = 1$. The inflection points ($\mu \pm \sigma$) are at 49 and 51 and the $\mu \pm 2\sigma$ landmarks are 48 and 52.

A sketch of the sampling distribution of \bar{X} :



- (b) Yes, a sample mean of 47 would be surprising because it is $3\sigma_{\bar{x}}$ units below the mean, so it is an unlikely observation.
- (c) A sample mean of 51.5 would not be very surprising, since it is $1.5\sigma_{\bar{x}}$ units to the right of center. This would not be too unusual under the stated conditions.