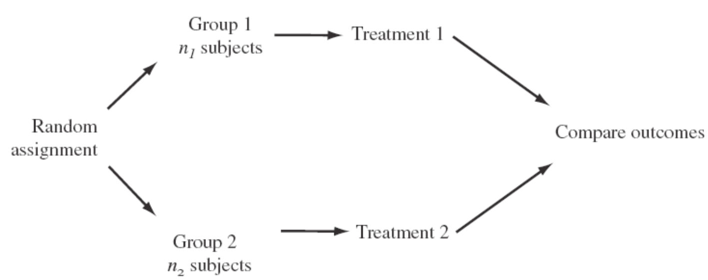
Formulas and Tables for Gerstman

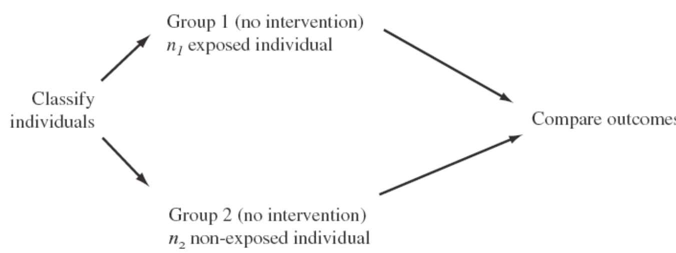
Measurement and Study Design

- Biostatistics is more than a compilation of computational techniques!
- Measurement scales: quantitative, ordinal, categorical
- Information quality is primary (GIGO)
- Data table: observations, variables, values
- Use Table A or random number generator to choose simple random sample or randomize a treatment
- Comparative studies must strive for group comparability to make valid inferences. Comparative studies may be either experimental or observational in design:

Experimental



Non-experimental



- Beware lurking variables, esp. in non-experimental studies: confounding!

Exploring and Describing Data

- Explore distributional shape, location, and spread; check for outliers
- Frequency, relative frequency, cumulative frequency
- Sample mean: $\bar{x} = \frac{1}{n}\sum x_i$
- Median: Form an ordered array. The median is the value with a depth of $\frac{n+1}{2}$; when n is odd, average the two middle values**.**
- Quartiles (Tukey's hinges): Divide the ordered array at the median; when n is odd, the median belongs to both the low group and the high group. Q1 is median of the low group. Q3 is the median of the high group.
- IQR = Q3 − Q1
- Boxplot: plot median and quartiles (box); determine fences: F_L = Q1 − 1.5·IQR, F_U = Q3 + 1.5·IQR; plot outside values (if any); draw whiskers from hinges to inside values
- Five-point summary: minimum, Q1, median, Q3, maximum
- Sample standard deviation: $s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$

Probability

- Basic properties: (1) $0 \le \Pr(A) \le 1$; (2) $\Pr(S) = 1$; (3) $\Pr(\bar{A}) = 1 - \Pr(A)$
  (4) Pr(A or B) = Pr(A) + Pr(B) for disjoint events
- Binomial: $X \sim b(n, p)$, $\Pr(X = x) = {}_nC_x p^x q^{n-x}$ where ${}_nC_x = \dfrac{n!}{x!(n-x)!}$

  $\mu = np$ and $\sigma = \sqrt{npq}$ where $q = 1 - p$
- To find probabilities on $X \sim N(\mu, \sigma)$:
  (1) State (2) Standardize $z = \dfrac{x - \mu}{\sigma}$ (3) Sketch (4) Use Table B
- To find percentile values on $X \sim N(\mu, \sigma)$:
  (1) State (2) Sketch (3) Table B (4) Unstandardize: $x = \mu + z_p\sigma$

Sampling Distributions and Introduction to Inference

- Sampling distribution of mean: $\bar{x} \sim N(\mu, \sigma/\sqrt{n})$ when population Normal or sample large
- Hypothesis testing procedure:
  (A.) H_0 and H_a (B.) Test statistic (C.) *P*-value (D.) *Optional*: Significance level
- One-sample test of mean (σ known): H_0: $\mu = \mu_0$; $z = \dfrac{\bar{x} - \mu_0}{SE_{\bar{x}}}$ where $SE_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
- Confidence interval for μ (σ known): $\bar{x} \pm z_{1-\alpha/2} \cdot SE_{\bar{x}}$ where $SE_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
- Conditions for *z* procedures: SRS, Normal population or large sample, σ known

Inference about Means

- Single samples and match-pairs (matched-pairs, analyze the "delta variable")

  To test H_0: $\mu = \mu_0$ use $t_{stat} = \dfrac{\bar{x} - \mu_0}{SE_{\bar{x}}}$ where $SE_{\bar{x}} = \dfrac{s}{\sqrt{n}}$ and $df = n - 1$

  $(1 - \alpha)100\%$ Confidence interval for μ: $\bar{x} \pm t_{n-1, 1-\frac{\alpha}{2}} \cdot SE_{\bar{x}}$

  Sample size and power to limit margin of error *m*, use $n = \left(z_{1-\frac{\alpha}{2}} \dfrac{\sigma}{m} \right)^2$

  Sample size to detect a difference Δ with stated power and α, use $n = \dfrac{\sigma^2 \left(z_{1-\beta} + z_{1-\frac{\alpha}{2}}\right)^2}{\Delta^2}$

  Power of a test to detect Δ at given α: $1 - \beta = \Phi\left(-z_{1-\frac{\alpha}{2}} + \dfrac{|\Delta|\sqrt{n}}{\sigma} \right)$
- Two independent samples

  To test H_0: $\mu_1 = \mu_2$ use $t_{\mathbf{stat}} = \dfrac{\bar{x}_1 - \bar{x}_2}{SE_{\bar{x}_1 - \bar{x}_2}}$ where $SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

  $df_{\text{conservative}}$ = smaller of $(n_1 - 1)$ or $(n_2 - 1)$ [use df_{Welch} when working with a computer]

  $(1 - \alpha)100\%$ CI for $\mu_1 - \mu_2$ =: $(\bar{x}_1 - \bar{x}_2) \pm (t_{df, 1-\frac{\alpha}{2}})(SE_{\bar{x}_1 - \bar{x}_2})$

  To estimate $\mu_1 - \mu_2$ with margin of error *m*, use $n = \dfrac{2\sigma^2 z_{1-\frac{\alpha}{2}}^2}{m^2}$ in each group

  Sample size test H_0: $\mu_1 = \mu_2$ at given $1-\beta$ and α: use $n = \dfrac{2\sigma^2 \left(z_{1-\beta} + z_{1-\frac{\alpha}{2}}\right)^2}{\Delta^2}$ in each group

If not equal group sizes are not possible, calculate n, fix n_1, and use $n_2 = \frac{nn_1}{2n_1 - n}$

- k independent samples

  To test H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$: $F_{\text{stat}} = \frac{MSB}{MSW}$ with df_B and df_W via ANOVA table

Variance	Sum of Squares	df	Mean Square
Between groups	$SS_B = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{x})^2$	$df_B = k - 1$	$MSB = \frac{SS_B}{df_B}$
Within groups	$SS_W = \sum_{i=1}^{k} (n_i - 1)s_i^2$	$df_W = N - k$	$MSW = \frac{SS_W}{df_W}$
Total	$SS_T = SS_B + SS_W$	$df = df_B + df_W$	

- Post hoc least square difference; $t_{\text{stat}} = \frac{\bar{x}_i - \bar{x}_j}{SE_{\bar{x}_i - \bar{x}_j}}$ where $SE_{\bar{x}_i - \bar{x}_j} = \sqrt{MSW\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$; $df = N - k$

  Confidence interval for $\mu_i - \mu_j = (\bar{x}_1 - \bar{x}_2) \pm (t_{df,1-\frac{\alpha}{2}})(SE_{\bar{x}_1 - \bar{x}_2})$

- Post hoc Bonferroni: multiply P by number of comparisons and use $t_{N-k,1-\frac{\alpha}{2c}}$ in the CI formula

Correlation and Regression (use calculator or computer)

- Correlation coefficient $r = \frac{1}{n-1}\sum z_X z_Y$

  To test H_0: $\rho = 0$, use $t_{\text{stat}} = \frac{r}{SE_r}$ where $SE_r = \sqrt{\frac{1-r^2}{n-2}}$ and $df = n - 2$

  Confidence interval for ρ: $LCL = \frac{r - \varpi}{1 - r\varpi}$ and $UCL = \frac{r + \varpi}{1 + r\varpi}$ where $\varpi = \sqrt{\frac{t^2_{df,1-\frac{\alpha}{2}}}{t^2_{df,1-\frac{\alpha}{2}} + df}}$

- Regression line: $\hat{y} = a + bx$

  Slope estimate: $b = r\frac{s_Y}{s_X}$

  Intercept estimate: $a = \bar{y} - b\bar{x}$

  Standard error of the regression $s_{Y|x} = \sqrt{\frac{1}{n-2}\sum \text{residuals}^2}$ with $df = n - 2$

  $(1 - \alpha)100\%$ confidence interval for $\beta = b \pm (t_{n-2,1-\alpha/2})(SE_b)$ where $SE_b = \frac{s_{Y|x}}{\sqrt{n-1} \cdot s_X}$

  To test H_0: $\beta = 0$, use $t_{\text{stat}} = \frac{b}{SE_b}$ with $df = n - 2$

- Multiple regression model: $\hat{y} = a + b_1x_1 + b_2x_2 + \cdots + b_kx_k$ (determine regression coefficients with computer program)

Inference about Proportions

Single sample

- Sample proportion: $\hat{p} = \dfrac{\text{number of successes}}{n}$
- $(1-\alpha)100\%$ CI for $p = \tilde{p} \pm z_{1-\frac{\alpha}{2}} \cdot \sqrt{\tilde{p}\tilde{q} / \tilde{n}}$ where $\tilde{p} = \dfrac{x+2}{n+4}$ and $\tilde{q} = 1 - \tilde{p}$
- To test H_0: $p = p_0$ use $z_{stat} = \dfrac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$ (use exact binomial procedure in small samples)
- Sample size requirement to limit the margin of error (m): use $n = \dfrac{z^2_{1-\frac{\alpha}{2}} p^* q^*}{m^2}$

Two independent samples

- Notation (2-by-2 cross-tabulation)

	+	−	Total
Exposed	a_1	b_1	n_1
Non-exposed	a_2	b_2	n_2
Total	m_1	m_2	N

- Sample proportions (cohort and naturalistic samples)
  $\hat{p}_1 = \dfrac{a_1}{n_1}$ and $\hat{p}_2 = \dfrac{a_2}{n_2}$

- To test H_0: $p_1 = p_2$, (z_{stat} and X^2_{stat} are equivalent for 2-by-2)
  $z_{\text{stat}} = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\bar{p}\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $\bar{p} = \dfrac{\text{number of successes, both samples combined}}{\text{total observations, both samples combined}}$

  $X^2_{\text{stat}} = \sum_{\text{all}} \left[\dfrac{(O_i - E_i)^2}{E_i} \right]$ where $E_i = \dfrac{\text{row total} \times \text{column total}}{\text{table total}}$ with $df = (R-1)(C-1)$

  In small samples, use Fisher's exact test or the Mid-P modification (computer)

- Risk difference = $\hat{p}_1 - \hat{p}_2$ (do not use in case-control sample)
  $(1-\alpha)100\%$ CI $p_1 - p_2 = = (\tilde{p}_1 - \tilde{p}_2) \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\tilde{p}_1 - \tilde{p}_2}$
  where $\tilde{p}_i = \dfrac{a_i + 1}{n_i + 2}$ and $SE_{\tilde{p}_1 - \tilde{p}_2} = \sqrt{\dfrac{\tilde{p}_1\tilde{q}_1}{\tilde{n}_1} + \dfrac{\tilde{p}_2\tilde{q}_2}{\tilde{n}_2}}$

- Risk ratio = $\hat{RR} = \dfrac{\hat{p}_1}{\hat{p}_2} = \dfrac{a_1 / n_1}{a_2 / n_2}$ (do not use in case-control sample)
  $(1-\alpha)100\%$ CI for $RR = e^{\ln \hat{RR} \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\ln \hat{RR}}}$ where $SE_{\ln \hat{RR}} = \sqrt{\dfrac{1}{a_1} - \dfrac{1}{n_1} + \dfrac{1}{a_2} - \dfrac{1}{n_2}}$

- Odds ratio = $\hat{O}R = \dfrac{a_1 \cdot b_2}{a_2 \cdot b_1}$

  $(1-\alpha)100\%$ CI for the $OR = e^{\ln \hat{O}R \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\ln \hat{O}R}}$ where $SE_{\ln \hat{O}R} = \sqrt{\dfrac{1}{a_1} + \dfrac{1}{b_1} + \dfrac{1}{a_2} + \dfrac{1}{b_2}}$

Matched-pairs

- Notation (for case-control data)

	Case E+	Case E−
Control E+	*a*	*b*
Control E−	*c*	*d*

- $\hat{O}R = \dfrac{c}{b}$
- $(1-\alpha)100\%$ CI for the $OR = e^{\ln \hat{O}R \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\ln \hat{O}R}}$ where $SE_{\ln \hat{O}R} = \sqrt{\dfrac{1}{c} + \dfrac{1}{b}}$
- To test H_0: $OR = 1$, use $z_{stat} = \sqrt{\dfrac{(c-b)^2}{c+b}}$

Table A. Two thousand random digits

Line										
01	79587	19407	49825	58687	99639	82670	73457	53546	30292	75741
02	02213	54407	22917	67392	51745	53341	74452	66258	19597	38440
03	77633	43390	63003	55825	63714	40243	91576	90982	71540	04987
04	02927	39916	38879	97492	54232	26582	75594	31430	62481	48852
05	27673	08260	19904	22537	85260	03805	27138	83323	82080	65863
06	53302	10918	20917	50444	34147	78213	19541	55366	81300	98651
07	43372	88167	59836	05054	51874	59309	72740	58205	60603	55196
08	67893	05723	37080	64029	75438	13959	16442	50847	33442	99647
09	54532	47973	68704	47487	29668	31437	11068	11238	24304	15632
10	34867	89777	96947	44092	49866	94813	71694	78305	33524	30622
11	95162	43739	48362	85438	70133	18178	56655	48265	53784	36693
12	70230	91840	05955	30586	13850	24182	88039	16226	03304	28002
13	19551	63026	59709	55085	18293	50503	75710	24402	62411	97615
14	31237	82396	46680	94704	69287	24926	38249	68858	62146	00131
15	76931	95289	55809	19381	56686	37898	36275	15881	98125	55618
16	88630	59115	76942	53000	89109	61901	55927	96619	34893	97543
17	50728	87768	16193	90514	58042	64398	18491	96407	97303	93459
18	52677	87418	65211	04353	71242	43041	24940	59906	61926	36837
19	04247	38798	73286	99890	09907	17260	04619	47185	71470	98872
20	76012	83064	66743	58110	49524	51685	51815	11837	06368	68488
21	77403	60931	68951	69023	02578	08934	89067	96693	07387	94489
22	70045	45404	80652	60568	94238	08517	34838	60958	94947	98568
23	74424	09905	65366	62295	26118	87077	19265	97192	45317	67620
24	39950	05637	14388	10366	67923	29927	72973	55083	83840	45719
25	13510	32969	80172	86599	57381	52330	38380	28773	97261	75126
26	30108	01696	59451	01073	27760	86472	04865	51333	83736	52416
27	00982	91303	72173	72499	26938	78075	74684	98037	18851	11754
28	47948	47652	25224	65500	86080	47438	11404	56085	04416	22130
29	54985	64122	15648	24313	46612	28442	74549	69001	89813	61596
30	48786	26571	21652	54949	57714	05975	82721	05667	13121	31879
31	81552	66957	51926	54171	50576	41745	87903	80302	76901	18060
32	99551	02072	20173	01563	01602	63964	59429	81601	74924	87038
33	67881	88556	16382	85038	67970	31366	67243	78854	63456	16789
34	06162	40256	69688	98904	82391	82920	13214	25743	31805	82401
35	63716	64311	26224	94569	18043	26137	99795	19047	92258	95604
36	46558	56764	32508	81263	43490	29181	38375	99015	37766	52912
37	14202	65556	24283	65881	37766	54388	80069	78335	79539	60511
38	53265	52355	13913	22834	95995	24878	14148	60663	03207	95208
39	15815	72512	32388	93730	31126	11194	91331	19052	64565	87124
40	56492	44200	29678	29214	08990	01549	40625	52756	62466	96748

Table B: Cumulative probabilities for a Standard Normal *Z* variable; traditional *z* table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table C. Traditional *t* table; tables entries represent *t* values.

Cumulative / *Upper tail*	0.75 / 0.25	0.80 / 0.20	0.85 / 0.15	0.90 / 0.10	0.95 / 0.05	0.975 / 0.025	0.99 / 0.01	0.995 / 0.005	0.9975 / 0.0025	0.999 / 0.001	0.9995 / 0.0005
df 1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
10	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞ (*z*)	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.5%**	**99.8%**	**99.9%**

Confidence level: (1 – α/2)100%

Table E. Chi-square table

	Upper tail *P*										
df	.975	.25	.20	.15	.10	.05	.025	.01	.01	.001	.0005
1	0.001	1.32	1.64	2.07	2.71	3.84	5.02	6.63	7.88	10.83	12.12
2	0.051	2.77	3.22	3.79	4.61	5.99	7.38	9.21	10.60	13.82	15.20
3	0.216	4.11	4.64	5.32	6.25	7.81	9.35	11.34	12.84	16.27	17.73
4	0.48	5.39	5.99	6.74	7.78	9.49	11.14	13.28	14.86	18.47	20.00
5	0.83	6.63	7.29	8.12	9.24	11.07	12.83	15.09	16.75	20.52	22.11
6	1.24	7.84	8.56	9.45	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	1.69	9.04	9.80	10.75	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	2.18	10.22	11.03	12.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	2.70	11.39	12.24	13.29	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	3.25	12.55	13.44	14.53	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	3.82	13.70	14.63	15.77	17.28	19.68	21.92	24.72	26.76	31.26	33.14
12	4.40	14.85	15.81	16.99	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	5.01	15.98	16.98	18.20	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	5.63	17.12	18.15	19.41	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	6.26	18.25	19.31	20.60	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	6.91	19.37	20.47	21.79	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	7.56	20.49	21.61	22.98	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	8.23	21.60	22.76	24.16	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	8.91	22.72	23.90	25.33	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	9.59	23.83	25.04	26.50	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	10.28	24.93	26.17	27.66	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	10.98	26.04	27.30	28.82	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	11.69	27.14	28.43	29.98	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	12.40	28.24	29.55	31.13	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	13.12	29.34	30.68	32.28	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	13.84	30.43	31.79	33.43	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	14.57	31.53	32.91	34.57	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	15.31	32.62	34.03	35.71	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	16.05	33.71	35.14	36.85	39.09	42.56	45.72	49.59	52.34	58.30	60.7
30	16.79	34.80	36.25	37.99	40.26	43.8	47.0	50.9	53.7	59.7	62.2
40	24.43	45.62	47.27	49.24	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	32.36	56.33	58.16	60.35	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	40.48	66.98	68.97	71.34	74.40	79.08	83.30	88.38	91.95	99.61	102.69
80	57.15	88.13	90.41	93.11	96.58	101.88	106.63	112.33	116.32	124.84	128.26
100	74.22	109.1	111.7	114.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

Table F: Two tails of z; table entries are two-sided P-values for $|z|$.

$\|z\|$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	1.0000	.99202	.98404	.97607	.96809	.96012	.95216	.94419	.93624	.92829
0.1	.92034	.91241	.90448	.89657	.88866	.88076	.87288	.86501	.85715	.84931
0.2	.84148	.83367	.82587	.81809	.81033	.80259	.79486	.78716	.77948	.77182
0.3	.76418	.75656	.74897	.74140	.73386	.72634	.71885	.71138	.70395	.69654
0.4	.68916	.68181	.67449	.66720	.65994	.65271	.64552	.63836	.63123	.62413
0.5	.61708	.61005	.60306	.59611	.58920	.58232	.57548	.56868	.56191	.55519
0.6	.54851	.54186	.53526	.52869	.52217	.51569	.50925	.50286	.49650	.49019
0.7	.48393	.47770	.47152	.46539	.45930	.45325	.44725	.44130	.43539	.42953
0.8	.42371	.41794	.41222	.40654	.40091	.39533	.38979	.38430	.37886	.37347
0.9	.36812	.36282	.35757	.35237	.34722	.34211	.33706	.33205	.32709	.32217
1.0	.31731	.31250	.30773	.30301	.29834	.29372	.28914	.28462	.28014	.27571
1.1	.27133	.26700	.26271	.25848	.25429	.25014	.24605	.24200	.23800	.23405
1.2	.23014	.22628	.22246	.21870	.21498	.21130	.20767	.20408	.20055	.19705
1.3	.19360	.19020	.18684	.18352	.18025	.17702	.17383	.17069	.16759	.16453
1.4	.16151	.15854	.15561	.15272	.14987	.14706	.14429	.14156	.13887	.13622
1.5	.13361	.13104	.12851	.12602	.12356	.12114	.11876	.11642	.11411	.11183
1.6	.10960	.10740	.10523	.10310	.10101	.09894	.09691	.09492	.09296	.09103
1.7	.08913	.08727	.08543	.08363	.08186	.08012	.07841	.07673	.07508	.07345
1.8	.07186	.07030	.06876	.06725	.06577	.06431	.06289	.06148	.06011	.05876
1.9	.05743	.05613	.05486	.05361	.05238	.05118	.05000	.04884	.04770	.04659
2.0	.04550	.04443	.04338	.04236	.04135	.04036	.03940	.03845	.03753	.03662
2.1	.03573	.03486	.03401	.03317	.03235	.03156	.03077	.03001	.02926	.02852
2.2	.02781	.02711	.02642	.02575	.02509	.02445	.02382	.02321	.02261	.02202
2.3	.02145	.02089	.02034	.01981	.01928	.01877	.01827	.01779	.01731	.01685
2.4	.01640	.01595	.01552	.01510	.01469	.01429	.01389	.01351	.01314	.01277
2.5	.01242	.01207	.01174	.01141	.01109	.01077	.01047	.01017	.00988	.00960
2.6	.00932	.00905	.00879	.00854	.00829	.00805	.00781	.00759	.00736	.00715
2.7	.00693	.00673	.00653	.00633	.00614	.00596	.00578	.00561	.00544	.00527
2.8	.00511	.00495	.00480	.00465	.00451	.00437	.00424	.00410	.00398	.00385
2.9	.00373	.00361	.00350	.00339	.00328	.00318	.00308	.00298	.00288	.00279
3.0	.00270	.00261	.00253	.00245	.00237	.00229	.00221	.00214	.00207	.00200
3.1	.00194	.00187	.00181	.00175	.00169	.00163	.00158	.00152	.00147	.00142
3.2	.00137	.00133	.00128	.00124	.00120	.00115	.00111	.00108	.00104	.00100
3.3	.00097	.00093	.00090	.00087	.00084	.00081	.00078	.00075	.00072	.00070
3.4	.00067	.00065	.00063	.00060	.00058	.00056	.00054	.00052	.00050	.00048
3.5	.00047	.00045	.00043	.00042	.00040	.00039	.00037	.00036	.00034	.00033
3.6	.00032	.00031	.00029	.00028	.00027	.00026	.00025	.00024	.00023	.00022
3.7	.00022	.00021	.00020	.00019	.00018	.00018	.00017	.00016	.00016	.00015
3.8	.00014	.00014	.00013	.00013	.00012	.00012	.00011	.00011	.00010	.00010
3.9	.00010	.00009	.00009	.00008	.00008	.00008	.00007	.00007	.00007	.00007

Table G: Two-sided *P*-values from *t* statistics ("non-traditional t table")

	df (1 to 15)														
|t|	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.937	0.929	0.927	0.925	0.924	0.924	0.923	0.923	0.923	0.922	0.922	0.922	0.922	0.922	0.922
0.2	0.874	0.860	0.854	0.851	0.849	0.848	0.847	0.846	0.846	0.845	0.845	0.845	0.845	0.844	0.844
0.3	0.814	0.792	0.784	0.779	0.776	0.774	0.773	0.772	0.771	0.770	0.770	0.769	0.769	0.769	0.768
0.4	0.758	0.728	0.716	0.710	0.706	0.703	0.701	0.700	0.698	0.698	0.697	0.696	0.696	0.695	0.695
0.5	0.705	0.667	0.651	0.643	0.638	0.635	0.632	0.631	0.629	0.628	0.627	0.626	0.625	0.625	0.624
0.6	0.656	0.609	0.591	0.581	0.575	0.570	0.567	0.565	0.563	0.562	0.561	0.560	0.559	0.558	0.557
0.7	0.611	0.556	0.534	0.523	0.515	0.510	0.507	0.504	0.502	0.500	0.498	0.497	0.496	0.495	0.495
0.8	0.570	0.508	0.482	0.469	0.460	0.454	0.450	0.447	0.444	0.442	0.441	0.439	0.438	0.437	0.436
0.9	0.533	0.463	0.434	0.419	0.409	0.403	0.398	0.394	0.392	0.389	0.387	0.386	0.384	0.383	0.382
1.0	0.500	0.423	0.391	0.374	0.363	0.356	0.351	0.347	0.343	0.341	0.339	0.337	0.336	0.334	0.333
1.1	0.470	0.386	0.352	0.333	0.321	0.313	0.308	0.303	0.300	0.297	0.295	0.293	0.291	0.290	0.289
1.2	0.442	0.353	0.316	0.296	0.284	0.275	0.269	0.264	0.261	0.258	0.255	0.253	0.252	0.250	0.249
1.3	0.417	0.323	0.284	0.263	0.250	0.241	0.235	0.230	0.226	0.223	0.220	0.218	0.216	0.215	0.213
1.4	0.395	0.296	0.256	0.234	0.220	0.211	0.204	0.199	0.195	0.192	0.189	0.187	0.185	0.183	0.182
1.5	0.374	0.272	0.231	0.208	0.194	0.184	0.177	0.172	0.168	0.165	0.162	0.159	0.158	0.156	0.154
1.6	0.356	0.251	0.208	0.185	0.170	0.161	0.154	0.148	0.144	0.141	0.138	0.136	0.134	0.132	0.130
1.7	0.339	0.231	0.188	0.164	0.150	0.140	0.133	0.128	0.123	0.120	0.117	0.115	0.113	0.111	0.110
1.8	0.323	0.214	0.170	0.146	0.132	0.122	0.115	0.110	0.105	0.102	0.099	0.097	0.095	0.093	0.092
1.9	0.308	0.198	0.154	0.130	0.116	0.106	0.099	0.094	0.090	0.087	0.084	0.082	0.080	0.078	0.077
2.0	0.295	0.184	0.139	0.116	0.102	0.092	0.086	0.081	0.077	0.073	0.071	0.069	0.067	0.065	0.064
2.1	0.283	0.171	0.127	0.104	0.090	0.080	0.074	0.069	0.065	0.062	0.060	0.058	0.056	0.054	0.053
2.2	0.272	0.159	0.115	0.093	0.079	0.070	0.064	0.059	0.055	0.052	0.050	0.048	0.046	0.045	0.044
2.3	0.261	0.148	0.105	0.083	0.070	0.061	0.055	0.050	0.047	0.044	0.042	0.040	0.039	0.037	0.036
2.4	0.251	0.138	0.096	0.074	0.062	0.053	0.047	0.043	0.040	0.037	0.035	0.034	0.032	0.031	0.030
2.5	0.242	0.130	0.088	0.067	0.054	0.047	0.041	0.037	0.034	0.031	0.030	0.028	0.027	0.025	0.025
2.6	0.234	0.122	0.080	0.060	0.048	0.041	0.035	0.032	0.029	0.026	0.025	0.023	0.022	0.021	0.020
2.7	0.226	0.114	0.074	0.054	0.043	0.036	0.031	0.027	0.024	0.022	0.021	0.019	0.018	0.017	0.016
2.8	0.218	0.107	0.068	0.049	0.038	0.031	0.027	0.023	0.021	0.019	0.017	0.016	0.015	0.014	0.013
2.9	0.211	0.101	0.063	0.044	0.034	0.027	0.023	0.020	0.018	0.016	0.014	0.013	0.012	0.012	0.011
3.0	0.205	0.095	0.058	0.040	0.030	0.024	0.020	0.017	0.015	0.013	0.012	0.011	0.010	0.010	0.009

	df (16 to 30)														
|t|	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.937	0.929	0.927	0.925	0.924	0.924	0.923	0.923	0.923	0.922	0.922	0.922	0.922	0.922	0.922
0.2	0.874	0.860	0.854	0.851	0.849	0.848	0.847	0.846	0.846	0.845	0.845	0.845	0.845	0.844	0.844
0.3	0.814	0.792	0.784	0.779	0.776	0.774	0.773	0.772	0.771	0.770	0.770	0.769	0.769	0.769	0.768
0.4	0.758	0.728	0.716	0.710	0.706	0.703	0.701	0.700	0.698	0.698	0.697	0.696	0.696	0.695	0.695
0.5	0.705	0.667	0.651	0.643	0.638	0.635	0.632	0.631	0.629	0.628	0.627	0.626	0.625	0.625	0.624
0.6	0.656	0.609	0.591	0.581	0.575	0.570	0.567	0.565	0.563	0.562	0.561	0.560	0.559	0.558	0.557
0.7	0.611	0.556	0.534	0.523	0.515	0.510	0.507	0.504	0.502	0.500	0.498	0.497	0.496	0.495	0.495
0.8	0.570	0.508	0.482	0.469	0.460	0.454	0.450	0.447	0.444	0.442	0.441	0.439	0.438	0.437	0.436
0.9	0.533	0.463	0.434	0.419	0.409	0.403	0.398	0.394	0.392	0.389	0.387	0.386	0.384	0.383	0.382
1.0	0.500	0.423	0.391	0.374	0.363	0.356	0.351	0.347	0.343	0.341	0.339	0.337	0.336	0.334	0.333
1.1	0.470	0.386	0.352	0.333	0.321	0.313	0.308	0.303	0.300	0.297	0.295	0.293	0.291	0.290	0.289
1.2	0.442	0.353	0.316	0.296	0.284	0.275	0.269	0.264	0.261	0.258	0.255	0.253	0.252	0.250	0.249
1.3	0.417	0.323	0.284	0.263	0.250	0.241	0.235	0.230	0.226	0.223	0.220	0.218	0.216	0.215	0.213
1.4	0.395	0.296	0.256	0.234	0.220	0.211	0.204	0.199	0.195	0.192	0.189	0.187	0.185	0.183	0.182
1.5	0.374	0.272	0.231	0.208	0.194	0.184	0.177	0.172	0.168	0.165	0.162	0.159	0.158	0.156	0.154
1.6	0.356	0.251	0.208	0.185	0.170	0.161	0.154	0.148	0.144	0.141	0.138	0.136	0.134	0.132	0.130
1.7	0.339	0.231	0.188	0.164	0.150	0.140	0.133	0.128	0.123	0.120	0.117	0.115	0.113	0.111	0.110
1.8	0.323	0.214	0.170	0.146	0.132	0.122	0.115	0.110	0.105	0.102	0.099	0.097	0.095	0.093	0.092
1.9	0.308	0.198	0.154	0.130	0.116	0.106	0.099	0.094	0.090	0.087	0.084	0.082	0.080	0.078	0.077
2.0	0.295	0.184	0.139	0.116	0.102	0.092	0.086	0.081	0.077	0.073	0.071	0.069	0.067	0.065	0.064
2.1	0.283	0.171	0.127	0.104	0.090	0.080	0.074	0.069	0.065	0.062	0.060	0.058	0.056	0.054	0.053
2.2	0.272	0.159	0.115	0.093	0.079	0.070	0.064	0.059	0.055	0.052	0.050	0.048	0.046	0.045	0.044
2.3	0.261	0.148	0.105	0.083	0.070	0.061	0.055	0.050	0.047	0.044	0.042	0.040	0.039	0.037	0.036
2.4	0.251	0.138	0.096	0.074	0.062	0.053	0.047	0.043	0.040	0.037	0.035	0.034	0.032	0.031	0.030
2.5	0.242	0.130	0.088	0.067	0.054	0.047	0.041	0.037	0.034	0.031	0.030	0.028	0.027	0.025	0.025
2.6	0.234	0.122	0.080	0.060	0.048	0.041	0.035	0.032	0.029	0.026	0.025	0.023	0.022	0.021	0.020
2.7	0.226	0.114	0.074	0.054	0.043	0.036	0.031	0.027	0.024	0.022	0.021	0.019	0.018	0.017	0.016
2.8	0.218	0.107	0.068	0.049	0.038	0.031	0.027	0.023	0.021	0.019	0.017	0.016	0.015	0.014	0.013
2.9	0.211	0.101	0.063	0.044	0.034	0.027	0.023	0.020	0.018	0.016	0.014	0.013	0.012	0.012	0.011
3.0	0.205	0.095	0.058	0.040	0.030	0.024	0.020	0.017	0.015	0.013	0.012	0.011	0.010	0.010	0.009