

1. (9.2, p. 177)

(a)  $H_0 : \mu = 10$

$H_A : \mu \neq 10$

(b)  $H_0 : \mu = 8$

$H_A : \mu \neq 8$

(c)  $H_0 : \mu = 1$

$H_A : \mu \neq 1$

2. (9.6, p. 187)

$$p_{1\text{-sided}} = \Pr(Z > 1.72) = 0.0427$$

$$p_{2\text{-sided}} = 2 \times p_{1\text{-sided}} = 0.0854$$

3. (9.10, p. 187)

- We will test the hypothesis  $H_0 : \mu = 1.3$  against the two-sided alternative hypothesis  $H_A : \mu \neq 1.3$ .
- We set our significance level at  $\alpha = 0.05$  and calculate the following:
- $z_{\text{stat}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1.4 - 1.3}{0.3/\sqrt{25}} = 1.67$
- $p_{2\text{-sided}} = 2 \times p_{1\text{-sided}} = 2 \times \Pr(Z > z_{\text{stat}}) = 2 \times \Pr(Z > 1.67) = 2(0.0478) = 0.0956$
- Our  $p$ -value is greater than  $\alpha$ , so we fail to reject the null hypothesis.
- There is no evidence to support the claim that mean dose of lithium carbonate is significantly different from 1.3.
- Therefore, the sample mean is not significantly higher than that of a well-maintained patient population.

4. (9.12, p. 193) Let  $X$  represent gestational length such that  $X \sim N(39, 2)$ . Suppose  $\alpha = 0.05$ . Based on information from Problem 9.11, we have  $\sigma = 2$ ,  $n = 22$ , and  $\mu_0 = 39$ , where the hypotheses would be set up as

$$H_0 : \mu = 39$$

$$H_a : \mu \neq 39$$

Now, if the true mean gestational length for African American women is actually 38.5, then we have  $\mu_a = 38.5$ . We may now find the power of this test by plugging these pieces into our formula.

$$\begin{aligned}
 1 - \beta &= \Phi\left(-z_{1-\frac{\alpha}{2}} + \frac{|\mu_0 - \mu_a|\sqrt{n}}{\sigma}\right) \\
 &= \Phi\left(-z_{1-\frac{0.05}{2}} + \frac{|39 - 38.5|\sqrt{22}}{2}\right) \\
 &= \Phi\left(-z_{0.975} + \frac{|0.5|\sqrt{22}}{2}\right) \\
 &= \Phi\left(-1.96 + \frac{2.3452}{2}\right) \\
 &= \Phi(-0.7874) \approx \Phi(-0.79) \\
 &= \Pr(Z \leq -0.79) \\
 &= 0.2148 \text{ (from Table B)}
 \end{aligned}$$

Therefore, the power of this test is 0.2148. If the mean gestational length for African American women is truly  $\mu_a = 38.5$ , then we only have 0.2148 power to reject  $H_0$ .

5. (9.16, p. 194)

- We will test the hypothesis  $H_0 : \mu = 175$  against the two-sided alternative hypothesis  $H_A : \mu \neq 175$ .
- We set our significance level at  $\alpha = 0.05$  and calculate the following:
- $z_{\text{stat}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{195 - 175}{50/\sqrt{39}} = 2.4980$
- $p_{2\text{-sided}} = 2 \cdot p_{1\text{-sided}} = 2 \cdot \Pr(Z > z_{\text{stat}}) = 2 \cdot \Pr(Z > 2.4980) = 2(0.0062) = 0.0124$
- Our  $p$ -value is less than  $\alpha$ , therefore we reject the null hypothesis.
- We have sufficient evidence to conclude that among boys whose fathers had a heart attack, the mean fasting cholesterol level is significantly different that 175.
- Furthermore, since our  $z_{\text{stat}}$  is positive, we may say that this mean fasting cholesterol level is significantly higher than 175.

6. (10.2, p. 203)

- a) The margin of error of this estimate for exact 95% confidence is  
 $m = z_{1-\frac{\alpha}{2}}(SE_{\bar{x}}) = 1.96(0.22) = 0.4312$ .  
 (The approximate margin of error would be  $m \approx 2(SE_{\bar{x}}) = 2(0.22) = 0.44$ )
- b) 95% CI for  $\mu$ :  $\bar{x} \pm m = \bar{x} \pm z_{1-\frac{\alpha}{2}}(SE_{\bar{x}}) = 6.1 \pm 0.4312 = (5.67, 6.53)$  pounds.
- c) It means that 95% of ‘like intervals’ will capture  $\mu$  and 5% will not. Or, 95 out of 100 confidence intervals created in this manner will capture  $\mu$ .

d) We use the formula for  $SE_{\bar{x}}$  to calculate the population standard deviation:

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \Rightarrow \sigma = SE_{\bar{x}}(\sqrt{n}) = 0.22(\sqrt{81}) = 1.98 \text{ pounds}$$

7. (10.4, p. 203) We are given  $\sigma = 2$  lbs. For 99% confidence intervals, we have  $\alpha = 0.01$  so  $z_{1-\frac{\alpha}{2}} = z_{1-\frac{0.01}{2}} = z_{1-0.005} = z_{0.995} = 2.576$ .

a) 99% CI:  $\bar{x} \pm z_{1-\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right) = 6.1 \pm 2.576 \left( \frac{2}{\sqrt{81}} \right) = 6.1 \pm 0.5724 = (5.5276, 6.6724)$ .

With 99% confidence, we can say that the mean birth weight for babies in this population is between 5.53 and 6.67 pounds.

b) 99% CI:  $\bar{x} \pm z_{1-\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right) = 7.0 \pm 2.576 \left( \frac{2}{\sqrt{36}} \right) = 7.0 \pm 0.8587 = (6.1413, 7.8587)$ .

With 99% confidence, we can say that the mean birth weight for babies in this population is between 6.14 and 7.86 pounds.

c) 99% CI:  $\bar{x} \pm z_{1-\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right) = 5.8 \pm 2.576 \left( \frac{2}{\sqrt{9}} \right) = 5.8 \pm 1.7173 = (4.0827, 7.5173)$ .

With 99% confidence, we can say that the mean birth weight for babies in this population is between 4.08 and 7.52 pounds.

8. (10.8, p. 205) We will assume  $\sigma = 100$  g in the community and  $\alpha = 0.01$ .

a)  $n = \left( z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{m} \right)^2 = \left( z_{0.995} \cdot \frac{100}{10} \right)^2 = (2.576 \cdot 10)^2 = 663.58 \rightarrow n = 664$

b)  $n = \left( z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{m} \right)^2 = \left( z_{0.995} \cdot \frac{100}{25} \right)^2 = (2.576 \cdot 4)^2 = 106.17 \rightarrow n = 107$

9. (10.12, p. 207) Yes, the sample mean is significantly different than 7.2 pounds at  $\alpha = 0.05$  because the 95% confidence interval excludes that value.

That is, the sample mean weight of infants born to mothers who smoke is significantly different than the average birthweight of 7.2 pounds at  $\alpha = 0.05$ . From this information we can tell that infants of mothers who smoke tend to have lower birthweight than average.

10. (10.14, p. 208)

a) False – the confidence interval is centered on  $\bar{x}$  so 100% of the confidence intervals will capture  $\bar{x}$ .

b) False – 5% of the 95% confidence intervals will fail to capture  $\mu$ .

c) True