1. (11.8, p. 216)

   a) $df = 15$

   b) From Table C, $2.131 < t_{\text{stat}} < 2.602$

   c) From Table C, $0.01 < p_{\text{1-sided}} < 0.025$

   d) From Table C, $0.02 < p_{\text{2-sided}} < 0.05$

2. (11.22, p. 229)

   a) Not assigned.

   b)
      - $\bar{x} = 103.\bar{8}$
      - $s = 6.91817$
      - $n = 9 \Rightarrow df = 8$
      - $\alpha = 0.10 \Rightarrow t_{df,1-\alpha/2} = t_{8,0.95} = 1.860$
      - 95% CI for μ:

   $$\bar{x} \pm t_{8,0.95}\left(\frac{s}{\sqrt{n}}\right) = 103.8889 \pm 1.860\left(\frac{6.91817}{\sqrt{9}}\right) = 103.8889 \pm 4.2893 = (99.5996, 108.1782)$$

   c) From part b), we see that $m = t_{8,0.95}\left(\frac{s}{\sqrt{n}}\right) = 4.2893$.

   d) We use the formula on p. 226 to calculate

   $$n = \left(z_{1-\frac{\alpha}{2}}\frac{\sigma}{m}\right)^2 = \left(1.645 \cdot \frac{6.91817}{3}\right)^2 = (3.79346)^2 = 14.39 \to 15$$

   Since $n < 30$, we apply the adjustment factor with $df = n - 1 = 15 - 1 = 14$.

   $$f = \frac{df+3}{df+1} = \frac{17}{15} \quad\Longrightarrow\quad n = 15(f) = 15\left(\frac{17}{15}\right) = 17$$

   Note that an equivalent way of applying this rule is to simply add 2 to the total sample size whenever the first step of the calculations gives an n less than 30.

3. (11.24, p. 230)

   The description of the problem gives us that $H_0 : \mu_d = 0$, $H_A : \mu_d \neq 0$, $s = 5$ mmHg, $n = 36$ and $\alpha = 0.05$. We estimate $\sigma = 5$ mmHg based on s and calculate the power of the test to find a mean difference of $\Delta = 2.5$ mmHg.

$$\begin{aligned}
1-\beta &= \Phi\left(-z_{1-\frac{\alpha}{2}} + \frac{|\Delta|\sqrt{n}}{\sigma}\right) \\
&= \Phi\left(-z_{0.975} + \frac{|2.5|\sqrt{36}}{5}\right) \\
&= \Phi\left(-1.96 + \frac{15}{5}\right) \\
&= \Phi(1.04) \\
&= \Pr(Z \leq 1.04) \\
&= 0.8508 \quad \text{(from Table B)}
\end{aligned}$$

Therefore, the power of this test is 0.8508. If the mean difference in blood pressure before and after is truly given by $\Delta = 2.5$, then we had 0.8508 power to reject H_0.

4. (11.26, p. 231)

a) Using the rule $\Delta = QOL_{\text{BASE}} - QOL_{\text{3MO}}$, our differences are

$$\Delta = \{1, 3, 2, 1, 3, 4, 2, -1, 0, 2\}$$

b) Not assigned.

c)
- $\bar{x} = 1.7$
- $s = 1.494434$
- We test the null hypothesis $H_0 : \mu_d = 0$ against $H_A : \mu_d \neq 0$.
- We set our significance level at $\alpha = 0.05$.
- $t_{\text{stat}} = \dfrac{\bar{x} - 0}{s/\sqrt{n}} = \dfrac{1.7 - 0}{1.4944/\sqrt{10}} = 3.598$
- Using the T table and $df = 9$, we obtain

$$0.0025 < p_{\text{1-sided}} < 0.005 \implies 0.005 < p_{\text{2-sided}} < 0.01.$$

Equivalently, we may use software to exactly calculate $p_{\text{2-sided}} = 0.00577$.
- Since our range of p-values is less than our predetermined significance level α, we decide to reject the null hypothesis.
- We have strong evidence to conclude that quality of life at baseline is significantly different from quality of life after three months of treatment. In this case, since higher scores indicate a worse quality of life, and our t_{stat} is positive, we conclude that quality of life has significantly improved after three months of treatment.

*** Note that we may define Δ in the opposite way, where $\Delta = QOL_{\text{3MO}} - QOL_{\text{BASE}}$. In this situation, the scores given from part (a) would all be multiplied by -1, and $t_{\text{stat}} = -3.598$. Everything else remains the same. An interpreted conclusion could be modified to state that since higher scores indicate a worse quality of life, and our $t_{mboxstat}$ is negative, we see that quality of life was significantly worse before treatment (ie, significantly improved after three months of treatment).

5. (12.6, p. 249)

   (a) Since the question asks for the standard error of the mean difference **without assuming equal variances**, then we use the formula below (instead of the formula for pooled standard error).

$$SE_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{\frac{(32.37)^2}{5}+\frac{(41.73)^2}{6}} = \sqrt{499.79553} = 22.3561$$

   (b) $\bar{x}_1 - \bar{x}_2 = 219.4 - 163.83 = 55.57$, $t_{9,0.975} = 2.262 \implies$

$$\begin{aligned} 95\%\text{ CI} &= \bar{x}_1 - \bar{x}_2 \ \pm\ t_{9,0.975}\cdot SE_{\bar{x}_1-\bar{x}_2} \\ &= 55.57 \ \pm\ 2.262(22.3561) \\ &= (5.0005, 106.1395) \end{aligned}$$

   (c) $t_{\text{stat}} = \dfrac{\bar{x}_1-\bar{x}_2}{SE_{\bar{x}_1-\bar{x}_2}} = \dfrac{55.57}{22.3561} = 2.4857$

   $\rightarrow$ **Using $df_{\text{Welch}} = 9$**:
   From the t table,

   $2.262 < t_{\text{stat}} < 2.821 \ \Rightarrow\ 0.01 < p_{\text{1-sided}} < 0.025 \ \Rightarrow\ 0.02 < p_{\text{2-sided}} < 0.05$

   $\rightarrow$ **Using $df_{\text{Conserv}} = 4$**:
   From the t table,

   $2.132 < t_{\text{stat}} < 2.776 \ \Rightarrow\ 0.025 < p_{\text{1-sided}} < 0.05 \ \Rightarrow\ 0.05 < p_{\text{2-sided}} < 0.10$

6. (12.10, p. 254) Some cholesterol variable X has $\sigma = 40$ mg/dL. We want to conduct a 2-sided test of two group differences at $\alpha = 0.05$ with 80% power.

   a) Use the formula at bottom of p. 251

$$\begin{aligned} n &= \frac{2\sigma^2\left(z_{1-\beta}+z_{1-\alpha/2}\right)^2}{\Delta^2} \\ &= \frac{2(40)^2(0.84+1.96)^2}{10^2} \\ &= \frac{25088}{100} \\ &= 250.88 \end{aligned}$$

$$\implies \text{Take } n = 251$$

Therefore, we will take $n = 251$ subjects in each group, recruiting a total number of $2n = 2(251) = 502$ subjects.

b) Since we can only recruit $n_1 = 150$ subjects in Group 1, we solve for n_2, the number of subjects in Group 2, using the formula on p. 251 and $n = 251$ from part a).

$$n_2 = \frac{nn_1}{2n_1 - n} = \frac{251(150)}{2(150) - 251} = \frac{37650}{49} = 768.367 \quad \Longrightarrow \quad n_2 = 769$$

If we are restricted to $n_1 = 150$ subjects in Group 1, then we must recruit $n_2 = 769$ subjects in Group 2.

7. (12.12, p. 254)

a) Not assigned

b)
- $\bar{x}_1 - \bar{x}_2 = 0.31 - (-3.59) = 3.9$
- $SE_{\bar{x}_1 - \bar{x}_2} =$

$$= \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.298^2}{22} + \frac{2.506^2}{47}} = \sqrt{\frac{1.6848}{22} + \frac{6.2800}{47}}$$
$$= \sqrt{0.0766 + 0.1336} = \sqrt{0.2102} = 0.4585$$

→ **Using $df_{\textbf{Conserv}}$** $= \min(n_1, n_2) - 1 = n_1 - 1 = 22 - 1 = 21$:
- $\alpha = 0.05 \;\Rightarrow\; t_{df,1-\alpha/2} = t_{21,0.975} = 2.080$
- 95% CI for $\mu_1 - \mu_2$:

$$\bar{x}_1 - \bar{x}_2 \;\pm\; t_{21,0.975}(SE_{\bar{x}_1 - \bar{x}_2}) = 3.9 \pm 2.080(0.4585) = 3.9 \pm 0.9537 = (2.9463, 4.8537)$$

→ **Using $df_{\textbf{Welch}}$** $= 66.20305111$:
- $\alpha = 0.05 \;\Rightarrow\; t_{df,1-\alpha/2} = t_{66.2,0.975} = 1.9965$
- 95% CI for $\mu_1 - \mu_2$:

$$\bar{x}_1 - \bar{x}_2 \pm t_{66.2,0.975}(SE_{\bar{x}_1 - \bar{x}_2}) = 3.9 \pm 1.9965(0.4585) = 3.9 \pm 0.9154 = (2.9846, 4.8154)$$

This 95% CI suggests that average percent change in bone mineral content differs between women who are breastfeeding (Group 2) and women who are neither pregnant nor breastfeeding (Group 1). This is because the interval does not contain the value 0, which means that we would reject a 2-sided hypothesis test of equality between the group means at $\alpha = 0.05$.

Since the interval is entirely positive, and comparing the respective group means, the data suggests that women who are breastfeeding are experiencing greater losses of BMC.

8. (12.20, p. 258)

- We will the two-sided test of $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$.
- We set our significance level at $\alpha = 0.05$ and calculate the following:
- $$\begin{aligned} SE_{\bar{x}_1-\bar{x}_2} &= \\ &= \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{\frac{0.98^2}{337}+\frac{1.14^2}{370}} = \sqrt{\frac{0.9604}{337}+\frac{1.2996}{370}} \\ &= \sqrt{0.00285+0.00351} = \sqrt{0.00636} = 0.0797 \end{aligned}$$
- $t_{\text{stat}} = \dfrac{\bar{x}_1-\bar{x}_2}{SE_{\bar{x}_1-\bar{x}_2}} = \dfrac{1.6-1.64}{0.0797} = \dfrac{-0.04}{0.0797} = -0.5019$
- Two ways to calculate p-value
  - → **Using $df_{\text{Conserv}} = 336$**:
    From the t table, we conservatively round down to $df = 100$ and obtain $|t_{\text{stat}}| < 0.677 \Rightarrow p_{\text{1-sided}} > 0.25 \Rightarrow p_{\text{2-sided}} > 0.5$
    (or $p_{\text{exact}} = 0.6168$ with $df = 100$ or $p_{\text{exact}} = 0.6161$ with $df = 336$)
  - → **Using df_{Welch}** $= 702.69$:
    From the t table, we conservatively round down to $df = 100$ and obtain $|t_{\text{stat}}| < 0.677 \Rightarrow p_{\text{1-sided}} > 0.25 \Rightarrow p_{\text{2-sided}} > 0.5$
    (or $p_{\text{exact}} = 0.6168$ with $df = 100$ or $p_{\text{exact}} = 0.6159$ with $df = 702.69$)
- In both cases, our p-value is much greater than α, therefore we fail to reject the null hypothesis that mean duration of peak symptoms is the same for the treatment group and the control group.
- There is no evidence to suggest that the mean duration of peak symptoms in children with upper respiratory infections in the treatment (echinacea) group is significantly different than that of the control group.

******Some people used the pooled method for calculating the standard error of the difference between the means. In that case, the hypotheses, decision and conclusion statements are the same. The calculations that change are given below, keeping $\alpha = 0.05$.

- $s_p^2 = \frac{336(0.98)^2+369(1.14)^2}{336+369} = \frac{802.2468}{705} = 1.1379 \implies$
  $SE_p = \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{1.1379\left(\frac{1}{337}+\frac{1}{370}\right)} = \sqrt{0.00645} = 0.0803$
- $t_{\text{stat}} = \frac{\bar{x}_1-\bar{x}_2}{SE_p} = \frac{-0.04}{0.0803} = -0.4981$
- $df_p = n_1 + n_2 - 2 = 337 + 370 - 2 = 705$
- From the t table, we can conservatively round down to $df = 100$ and obtain $|t_{\text{stat}}| < 0.677 \Rightarrow p_{\text{1-sided}} > 0.25 \Rightarrow p_{\text{2-sided}} > 0.5$
  (or $p_{\text{exact}} = 0.6186$ with $df_p = 705$)