

1. (5.8) (p. 97)

(a) Let  $X$  be the age of mothers of newborns in the United States. Then,

$$\Pr(X \leq 19) = \Pr(X \leq 14) + \Pr(14 < X \leq 19) = 0.003 + 0.1 = 0.103$$

Thus, if we select a birth at random, the probability that the mother is less than or equal to 19 years of age is 0.103.

(b)  $\Pr(X \geq 30) = 0.3 + 0.097 = 0.397$

If we select a birth at random, the probability that the mother is at least 30 years of age is 0.397.

(c) The expected age of mothers of newborns in the U.S. is 28.91 years.

$$\begin{aligned} \mu &= \sum_x x \cdot \Pr(X = x) \\ &= 12.5(0.003) + 17.5(0.1) + 25(0.5) + 35(0.3) + 42.5(0.097) + 47.5(0.000) \\ &= 0.0375 + 1.75 + 12.5 + 10.5 + 4.1225 + 0 \\ &= 28.91 \text{ years} \end{aligned}$$

(d) The variance in age of mothers of newborns in the U.S. is 50.5119 years<sup>2</sup>.

$$\begin{aligned} \sigma^2 &= \sum_x (x - \mu)^2 \cdot \Pr(X = x) \\ &= (12.5 - 28.91)^2(0.003) + (17.5 - 28.91)^2(0.1) + (25 - 28.91)^2(0.5) + \\ &\quad + (35 - 28.91)^2(0.3) + (42.5 - 28.91)^2(0.097) + (47.5 - 28.91)^2(0.000) \\ &= 0.8079 + 13.0188 + 7.6441 + 11.1264 + 17.9147 + 0 \\ &= 50.5119 \text{ years}^2 \end{aligned}$$

2. (5.10) (p. 99)

$$\Pr(\text{individual self-identifies as Hispanic}) = 0.000 + 0.003 + 0.060 + 0.062 = 0.125$$

3. (5.12) (p. 103) - Refer to Figure 5.7

(a)  $\Pr(X \geq 0.6) = \text{base} \times \text{height} = (1 - 0.6)(1) = 0.4$

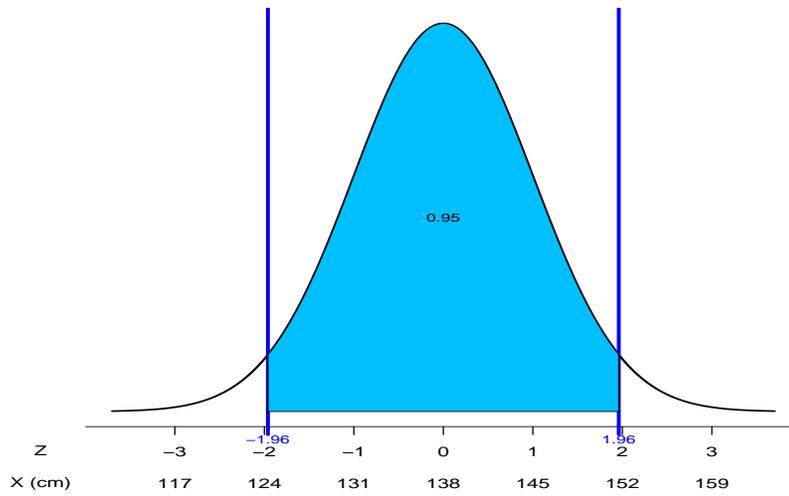
(b)  $\Pr(X \leq 0.6) = \text{base} \times \text{height} = (0.6)(1) = 0.6$

4. (7.2) (p. 139) Using the 68 – 95 – 99.7% approximation rule, we calculate the following.

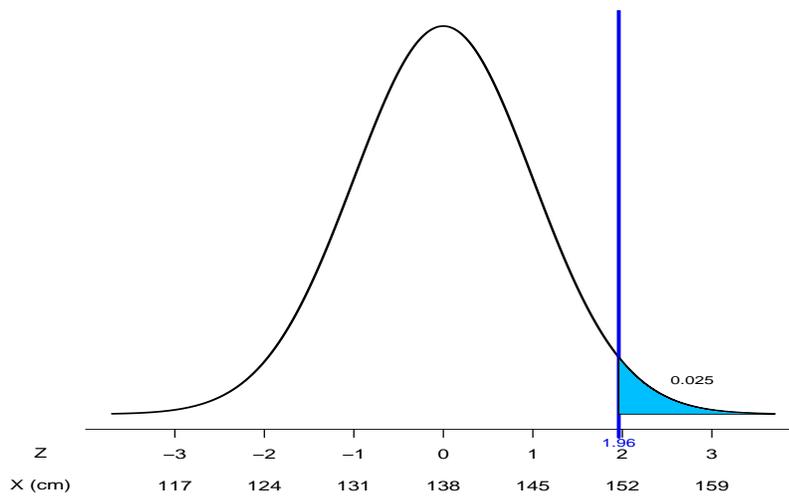
- 1) The middle 95% of heights are captured by the range  $\mu \pm 2(\sigma) = 138 \pm 2(7) = (124, 152)$  cm.
- 2) The tallest 2.5% are taller than  $\mu + 2(\sigma) = \mu + 2(7) = 152$  cm.

Alternatively, we can base the calculations on Table B (exact).

- 1) The middle 95% of heights are captured by the range  $\mu \pm 1.96(\sigma) = 138 \pm 1.96(7) = (124.28, 151.72)$  cm.

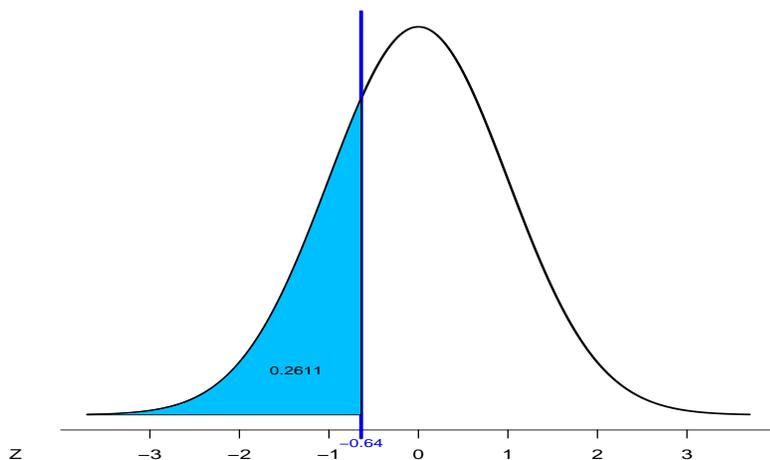


- 2) The tallest 2.5% are taller than  $\mu + 1.96(\sigma) = \mu + 1.96(7) = 151.72$  cm.

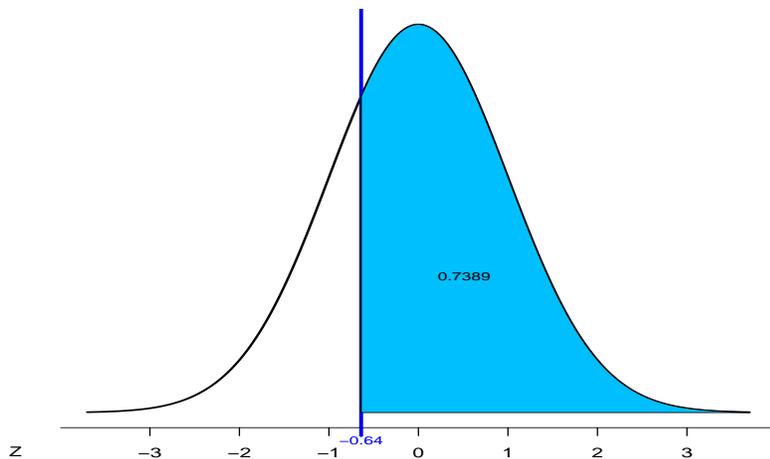


5. (7.4) (p. 143)

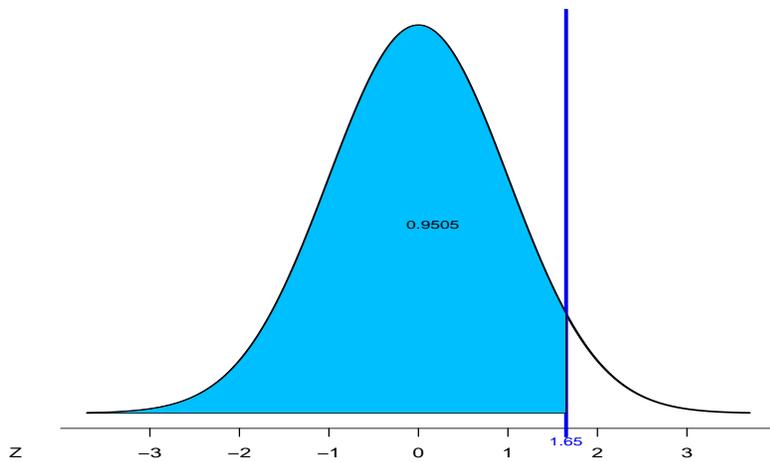
(a)  $\Pr(Z < -0.64) = 0.2611$



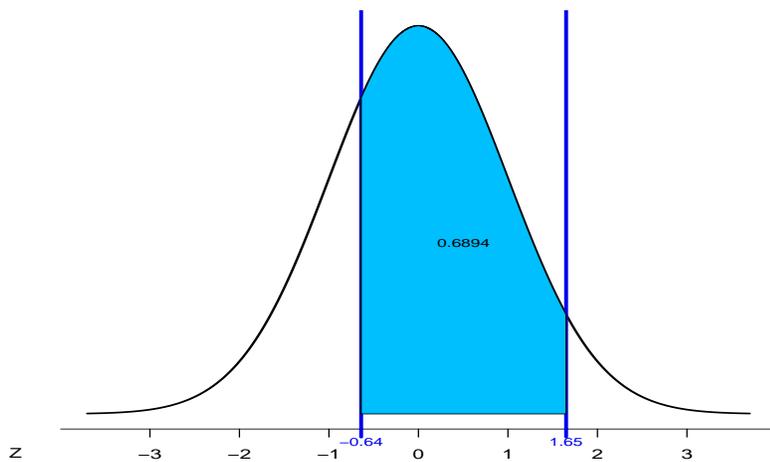
(b)  $\Pr(Z > -0.64) = 1 - 0.2661 = 0.7339$



$$(c) \Pr(Z < 1.65) = 0.9505$$



$$(d) \Pr(-0.64 < Z < 1.65) = \Pr(Z < 1.65) - \Pr(Z < -0.64) = 0.9505 - 0.2611 = 0.6894$$



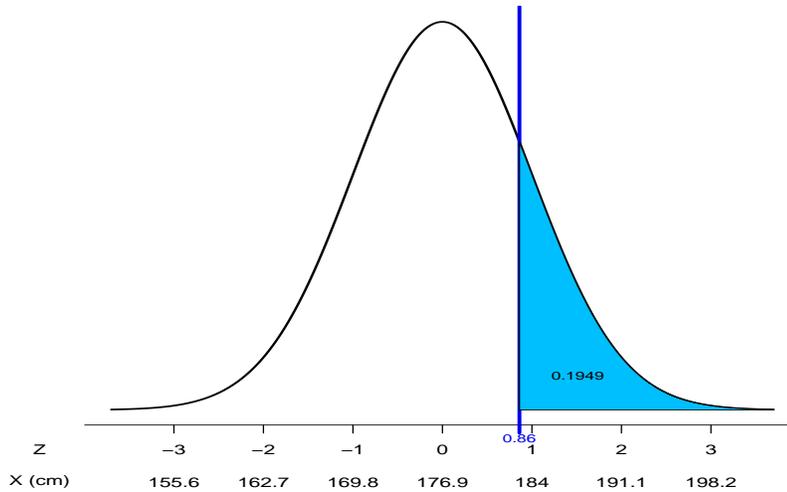
6. (7.6) (p. 145) - Sketch curves for parts a,b

(a) Let  $X$  represent the heights of 20 year old males such that  $X \sim N(176.9, 7.1)$ .

The quantity of interest is  $\Pr(X > 183)$ .

The standardized quantity is  $\Pr(Z > \frac{183-176.9}{7.1}) = \Pr(Z > 0.86)$ .

The sketch of this probability is



Thus from Table B,  $\Pr(Z > 0.86) = 0.1949$  (about 19.49%).

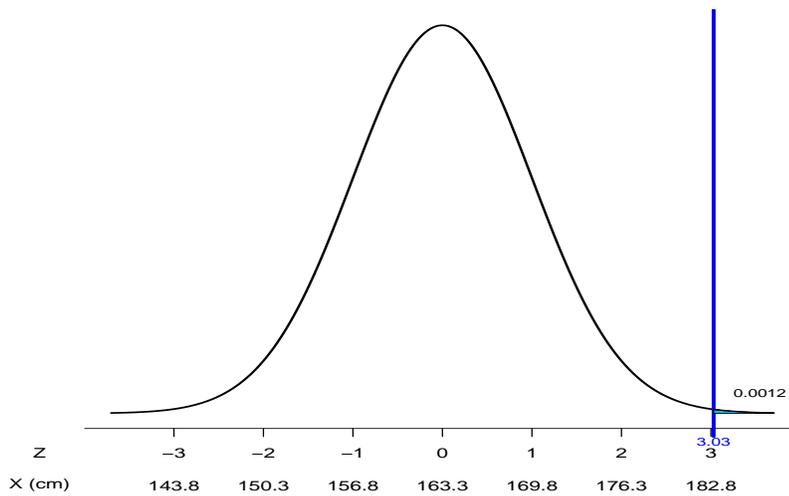
Therefore, 19.49% of U.S. men are at least 6 ft tall by age 20.

(b) Let  $Y$  be the height of 20 year old women such that  $Y \sim N(163.3, 6.5)$ .

The quantity of interest is  $\Pr(Y > 183)$ .

The standardized quantity is  $\Pr(Z > \frac{183-163.3}{6.5}) = \Pr(Z > 3.03)$ .

The sketch of the probability is



From Table B,  $\Pr(Z > 3.03) = 0.0012$ .

Therefore, only 0.12% of U.S. women are taller than 6 ft at age 20.

- (c) One out of every 5 20-year old U.S. men are at least 6 ft tall, while only 1 out of every 1000 20-year-old U.S. women are at least 6 ft tall.

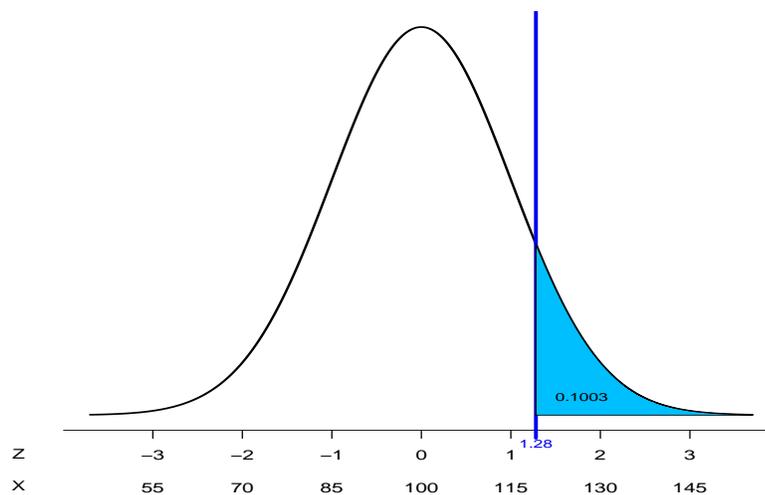
7. (7.10) (p. 147) - Sketch curves for both questions

Let  $X$  be the Wechsler Adult Intelligence Score such that  $X \sim N(100, 15)$ .

- 1) The quantity of interest is  $a$  such that  $\Pr(X > a) = 0.1$ . Equivalently, we want  $a$  such that  $\Pr(X \leq a) = 0.9$ .

The standardized quantity is  $a$  such that  $\Pr(Z \leq \frac{a-100}{15}) = 0.9$ .

The sketch of the probability is



From Table B,  $z_{0.9} = 1.28$ .

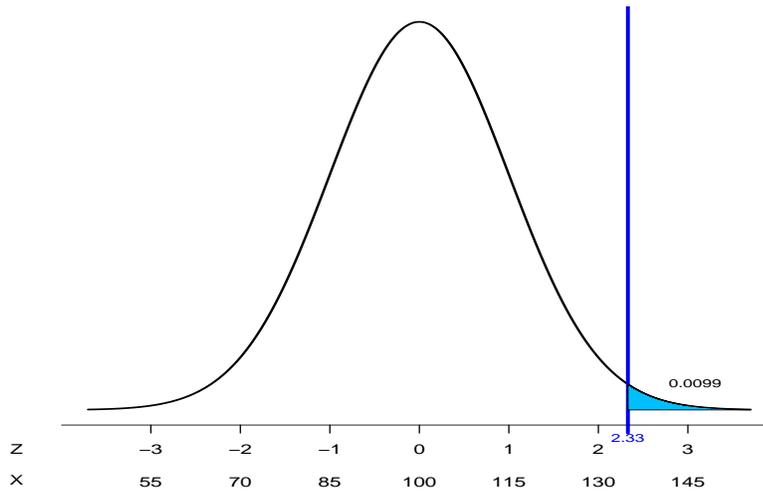
Unstandardization yields  $a = \mu + z_{0.9}\sigma = 100 + 1.28(15) = 119.2$ .

Therefore, scores above 119.2, or 120, are in the top 10% of scores.

- 2) The quantity of interest is  $a$  such that  $\Pr(X > a) = 0.01$ . Equivalently, we want  $a$  such that  $\Pr(X \leq a) = 0.99$ .

The standardized quantity is  $a$  such that  $\Pr(Z \leq \frac{a-100}{15}) = 0.99$ .

The sketch of the probability is



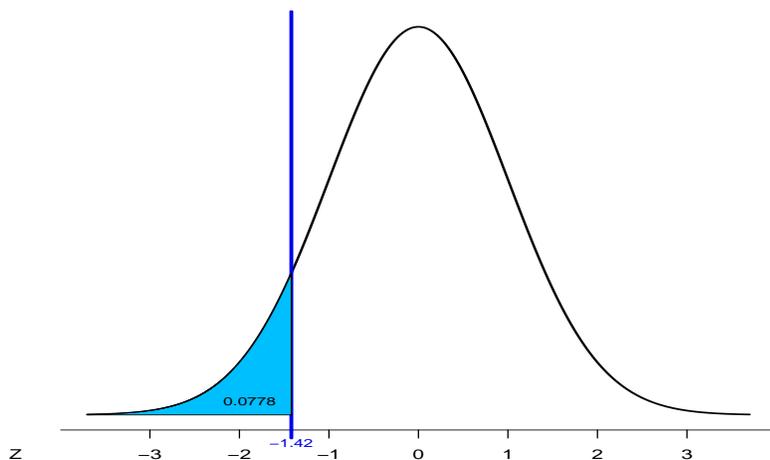
From Table B,  $z_{0.99} = 2.33$ .

Unstandardization yields  $a = \mu + z_{0.99}\sigma = 100 + 2.33(15) = 134.95$ .

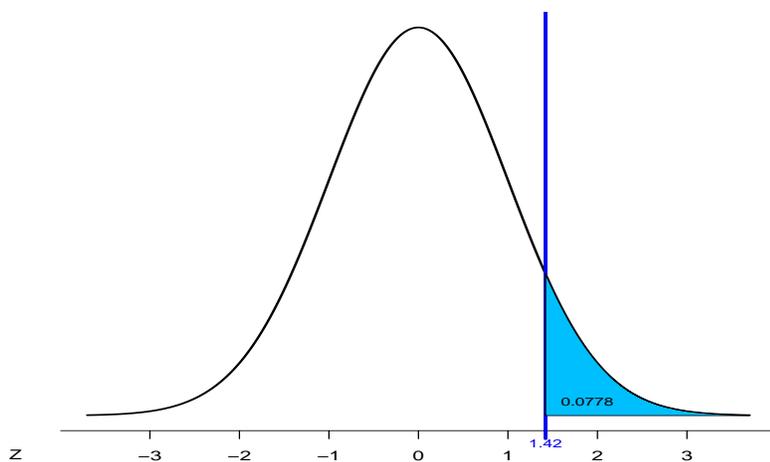
Therefore, scores above 134.95, or 135, are in the top 1% of scores.

8. (7.12) (p. 153) - Sketch curves

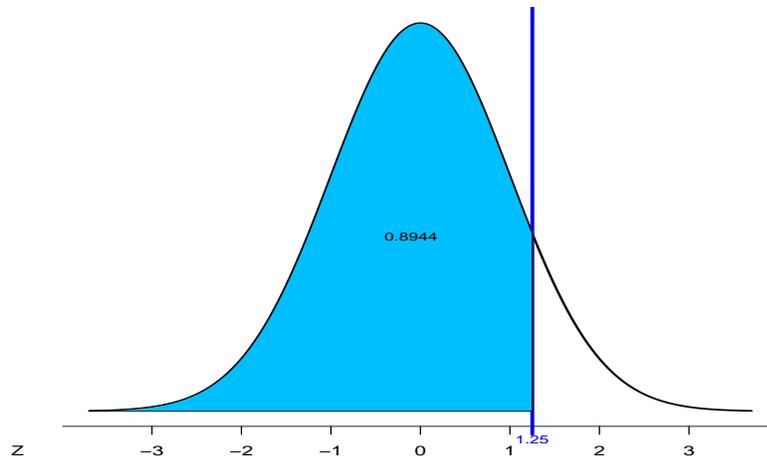
(a)  $\Pr(Z < -1.42) = 0.0778$



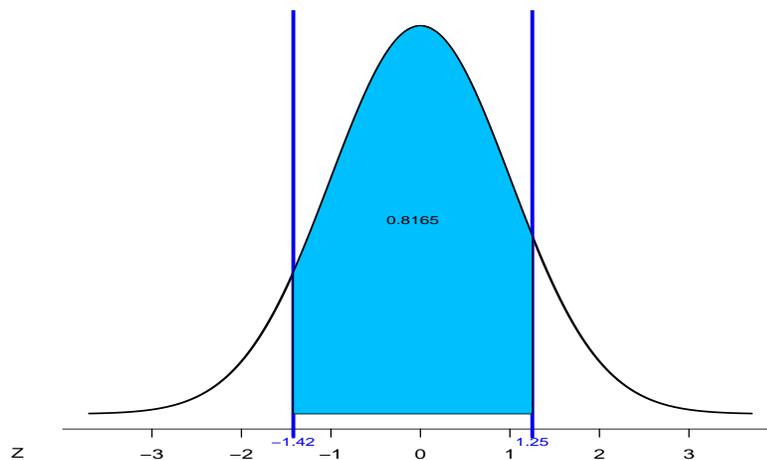
(b)  $\Pr(Z > 1.42) = 1 - \Pr(Z \leq 1.42) = 1 - 0.9222 = 0.0778$



(c)  $\Pr(Z < 1.25) = 0.8944$



(d)  $\Pr(-1.42 < Z < 1.25) = \Pr(Z < 1.25) - \Pr(Z < -1.42) = 0.8944 - 0.0778 = 0.8166$



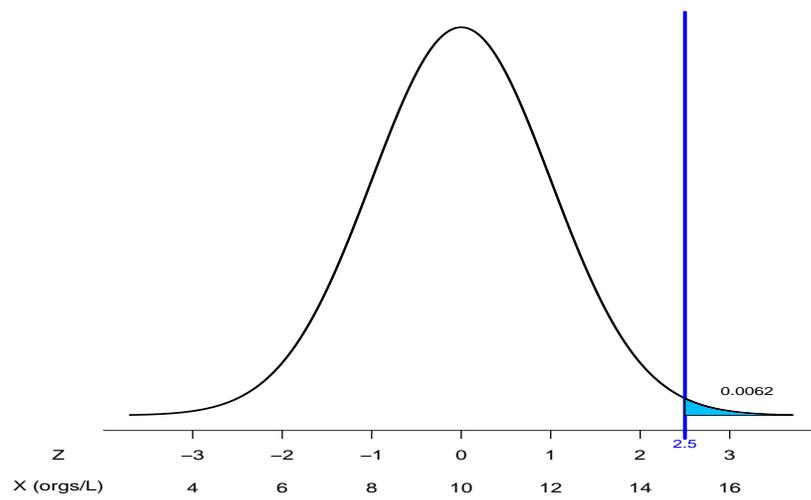
9. (7.14) (p. 153) - Sketch curve

Let  $X$  be the coliform level in a site such that  $X \sim N(10, 2)$ .

The quantity of interest is  $\Pr(X > 15)$ .

The standardized quantity is  $\Pr(Z > \frac{15-10}{\sqrt{2}}) = \Pr(Z > 2.5)$ .

The sketch of the probability is



From Table B,  $\Pr(Z > 2.5) = 1 - 0.9938 = 0.0062$ .

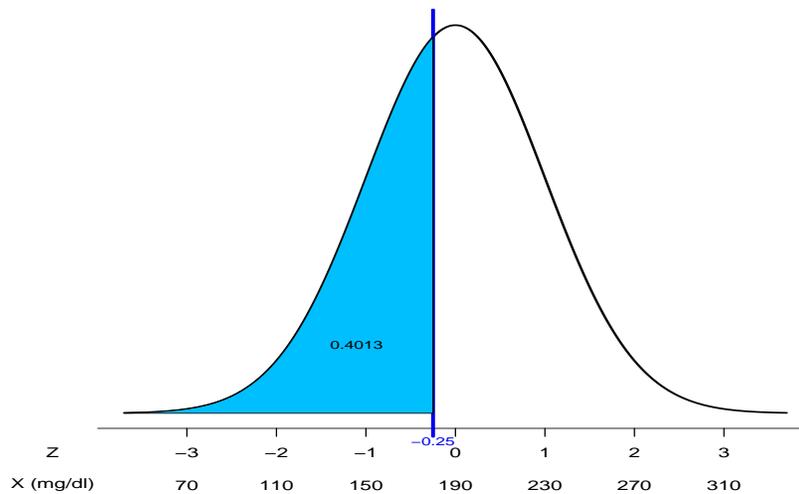
Therefore, there are 0.62% of samples with more than 15 organisms.

10. (8.6) (p. 166) Let  $X$  represent the serum cholesterol (mg/dl) in undergraduate men, such that  $X \sim N(190, 40)$ .

(a) The quantity of interest is  $\Pr(X \leq 180)$

The standardized quantity is  $\Pr(Z \leq -0.25)$

The sketch of the probability is



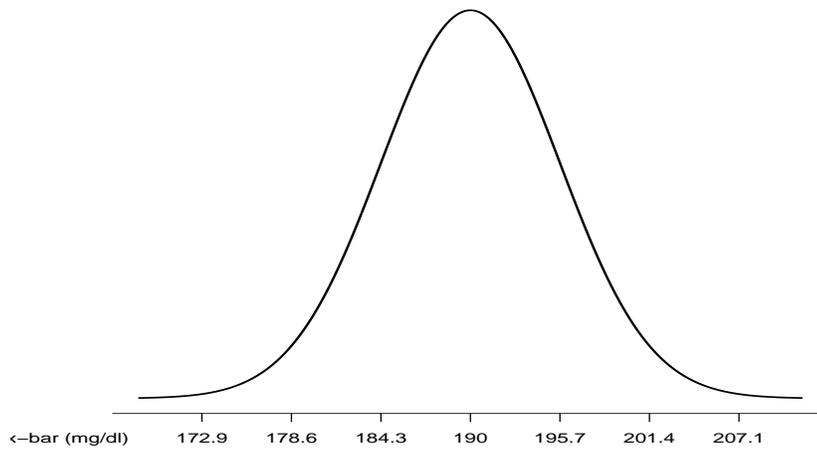
From Table B,  $\Pr(Z \leq -0.25) = 0.4013$

Therefore, the probability of randomly selecting an undergraduate man with serum cholesterol less than 180 is 0.4013.

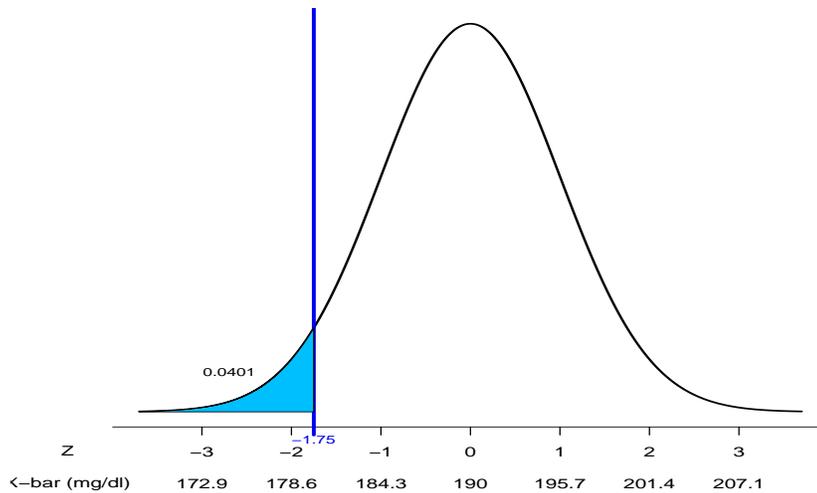
(b) To obtain the sampling distribution of  $\bar{x}$ , first calculate the standard deviation of  $\bar{x}$ .

$$\sigma_{\bar{X}} = \frac{40}{\sqrt{49}} = \frac{40}{7} = 5.7143$$

By Chapter 8 results, we know that  $\bar{X} \sim N\left(190, \frac{40}{7}\right)$ .



- (c) The quantity of interest is  $\Pr(\bar{X} \leq 180)$   
 The standardized quantity is  $\Pr(Z \leq -1.75)$   
 The sketch of the probability is



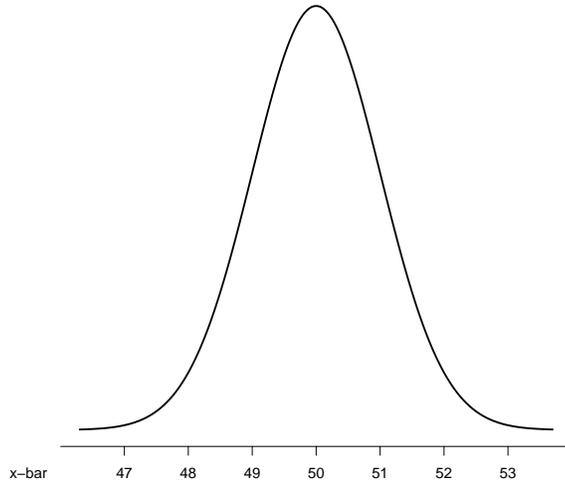
From Table B,  $\Pr(Z \leq -1.75) = 0.0401$

Therefore, the probability of getting a sample mean less than 180 is 0.0401.

11. (8.8) (p. 167)

- (a)  $\bar{X} \sim N\left(50, \frac{5}{\sqrt{25}}\right) \equiv N\left(50, \frac{5}{5}\right) \equiv N(50, 1)$ . That is, the sample mean  $\bar{X}$  is distributed Normally with mean  $\mu = 50$  and standard deviation  $SE_{\bar{X}} = 1$ . The inflection points ( $\mu \pm \sigma$ ) are at 49 and 51 and the  $\mu \pm 2\sigma$  landmarks are 48 and 52.

A sketch of the sampling distribution of  $\bar{X}$ :



- (b) Yes, a sample mean of 47 would be surprising because it is  $3\sigma_{\bar{x}}$  units below the mean, so it is an unlikely observation.
- (c) A sample mean of 51.5 would not be very surprising, since it is  $1.5\sigma_{\bar{x}}$  units to the right of center. This would not be too unusual under the stated conditions.