```
/*
     ARIMA: Models for Series J from Box-Jenkins
     REF: Box, G.E.P. and Jenkins, G.M. (1976),
     Time Series Analysis:  Forecasting and Control,
     San Francisco: Holden-Day.
*/
options ps=58 ls=80 pageno=1 nodate;
title1 'Gas Furnace Data';
title2 '(Box and Jenkins, Series J)';
data seriesj;
    input x y @@;
    label x = 'Input Gas Rate'
          y = 'Output CO2';
cards;
 -0.109  53.8  0.000  53.6  0.178  53.5  0.339  53.5
  0.373  53.4  0.441  53.1  0.461  52.7  0.348  52.4
  ...
  ...
  0.034  57.0  0.204  58.0  0.253  58.6  0.195  58.5
  0.131  58.3  0.017  57.8 -0.182  57.3 -0.262  57.0
 ;
proc arima data=seriesj;
/*
   Check the input series
*/
 i var=x nlags=10; run;

/*
   Fit a model for the input
*/
 e p=3; run;

/*
   Compute crosscorrelation of prewhitened series
*/
 i var=y crosscorr=(x) nlags=10; run;

/*
   Fit transfer function - examine residuals
*/
 e input=( 3$ (1,2)/(1,2) x ) plot; run;

/*
   Fit the final model
*/
```

```
e p=2 input=( 3$ (1,2)/(1) x ); run;
quit;
```

```
                       Gas Furnace Data                                  1
                  (Box and Jenkins, Series J)
                       ARIMA Procedure

               Name of variable = X.

               Mean of working series = -0.05683
               Standard deviation     = 1.070952
               Number of observations =      296

                       Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   1.146938    1.00000  |                    |********************|
  1   1.092430    0.95247  |                  . |******************* |
  2   0.956652    0.83409  |                .   |*****************   |
  3   0.782051    0.68186  |               .    |**************      |
  4   0.609291    0.53123  |               .    |***********         |
  5   0.467380    0.40750  |              .     |********            |
  6   0.364957    0.31820  |              .     |******              |
  7   0.298427    0.26019  |              .     |*****.              |
  8   0.260943    0.22751  |              .     |*****.              |
  9   0.244378    0.21307  |              .     |**** .              |
 10   0.238942    0.20833  |              .     |**** .              |

                   Partial Autocorrelations

           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.95247  |                  . |******************* |
             2   -0.78796  |    ****************| .                  |
             3    0.33897  |                  . |*******             |
             4    0.12121  |                  . |**                  |
             5    0.05896  |                  . |*.                  |
             6   -0.11147  |                  **| .                  |
             7    0.04862  |                  . |*.                  |
             8    0.09945  |                  . |**                  |
             9    0.01587  |                  . | .                  |
            10   -0.06973  |                  .*| .                  |
```

```
                Autocorrelation Check for White Noise

 To   Chi                  Autocorrelations
Lag  Square DF   Prob
  6  786.35  6  0.000  0.952  0.834  0.682  0.531  0.408  0.318
```

```
                 Conditional Least Squares Estimation

                                  Approx.
        Parameter   Estimate     Std Error   T Ratio  Lag
        MU          -0.12280       0.10902     -1.13   0
        AR1,1        1.97607       0.05499     35.94   1
        AR1,2       -1.37499       0.09967    -13.80   2
        AR1,3        0.34336       0.05502      6.24   3

        Constant Estimate  = -0.0068229

        Variance  Estimate = 0.03579652
        Std Error Estimate = 0.18919968
        AIC                = -141.66743*
        SBC                =   -126.906*
        Number of Residuals=        296
        * Does not include log determinant.

                    Correlations of the Estimates

   Parameter           MU       AR1,1       AR1,2       AR1,3

   MU               1.000      -0.017       0.014      -0.016
   AR1,1           -0.017       1.000      -0.941       0.790
   AR1,2            0.014      -0.941       1.000      -0.941
   AR1,3           -0.016       0.790      -0.941       1.000

                 Autocorrelation Check of Residuals

 To   Chi                  Autocorrelations
Lag  Square DF   Prob
  6   10.30  3  0.016 -0.042  0.068  0.056 -0.145 -0.009  0.059
 12   19.89  9  0.019  0.014  0.002 -0.055  0.035  0.143 -0.079
 18   27.92 15  0.022  0.099  0.043 -0.082  0.017  0.066 -0.052
 24   31.05 21  0.073 -0.078  0.024  0.015  0.030  0.045  0.004
```

```
  30   34.58 27  0.150 -0.007 -0.004  0.073 -0.038 -0.062  0.003
  36   38.84 33  0.223  0.010  0.002  0.082  0.045  0.056 -0.023
  42   41.18 39  0.375  0.002  0.033 -0.061 -0.003 -0.006 -0.043
  48   42.73 45  0.569  0.018  0.051 -0.012  0.015 -0.027  0.020

Model for variable X

Estimated Mean = -0.1228034

Autoregressive Factors
Factor 1: 1 - 1.9761 B**(1) + 1.375 B**(2) - 0.34336 B**(3)



                Name of variable = Y.

                Mean of working series = 53.50912
                Standard deviation     = 3.196707
                Number of observations =      296

                         Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0  10.218937    1.00000  |                    |********************|
  1   9.920101    0.97076  |                  . |******************* |
  2   9.156572    0.89604  |                .   |******************  |
  3   8.099002    0.79255  |               .    |****************    |
  4   6.948508    0.67996  |             .      |**************      |
  5   5.870550    0.57448  |             .      |***********         |
  6   4.960762    0.48545  |             .      |**********          |
  7   4.251890    0.41608  |             .      |********            |
  8   3.736119    0.36561  |            .       |*******             |
  9   3.376155    0.33038  |            .       |*******             |
 10   3.132316    0.30652  |            .       |******.             |

                     Partial Autocorrelations

           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.97076  |                  . |******************* |
             2    -0.80388  |    ****************| .                  |
             3     0.18833  |                  . |****                |
             4     0.25999  |                  . |*****               |
             5     0.05949  |                  . |*.                  |
             6    -0.06258  |                  .*| .                  |
```

```
                 7   -0.01435  |                  . | .                  |
                 8    0.05490  |                  . |*.                  |
                 9    0.00545  |                  . | .                  |
                10    0.03141  |                  . |*.                  |

                      Autocorrelation Check for White Noise

            To    Chi                  Autocorrelations
           Lag  Square DF   Prob
             6 1023.15  6  0.000   0.971  0.896  0.793  0.680  0.574  0.485



                        Correlation of Y and X
                        Both series have been prewhitened.
                        Variance of transformed series = 0.131438 and 0.035357
                        Number of observations =      296

                              Crosscorrelations

   Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
   -10  0.0015683    0.02301  |                  . | .                  |
    -9 0.00013502    0.00198  |                  . | .                  |
    -8 -0.0060480   -0.08872  |                  **| .                  |
    -7 -0.0017624   -0.02585  |                  .*| .                  |
    -6 -0.0080539   -0.11814  |                  **| .                  |
    -5 -0.0000944   -0.00138  |                  . | .                  |
    -4 -0.0012802   -0.01878  |                  . | .                  |
    -3 -0.0031078   -0.04559  |                  .*| .                  |
    -2 0.00065212    0.00957  |                  . | .                  |
    -1 -0.0019166   -0.02811  |                  .*| .                  |
     0 -0.0003673   -0.00539  |                  . | .                  |
     1  0.0038939    0.05712  |                  . |*.                  |
     2 -0.0016971   -0.02489  |                  . | .                  |
     3  -0.019231   -0.28210  |              ******| .                  |
     4  -0.022479   -0.32974  |             *******| .                  |
     5  -0.030909   -0.45341  |           *********| .                  |
     6  -0.018122   -0.26583  |               *****| .                  |
     7  -0.011426   -0.16761  |                 ***| .                  |
     8 -0.0017355   -0.02546  |                  .*| .                  |
     9  0.0022590    0.03314  |                  . |*.                  |
    10 -0.0035152   -0.05156  |                  .*| .                  |
```

Crosscorrelation Check Between Series

To Lag	Chi Square	DF	Prob	Crosscorrelations					
5	117.75	6	0.000	-0.005	0.057	-0.025	-0.282	-0.330	-0.453

Both variables have been prewhitened by the following filter:

Prewhitening Filter

Autoregressive Factors
Factor 1: 1 - 1.9761 B**(1) + 1.375 B**(2) - 0.34336 B**(3)

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag	Variable	Shift
MU	53.32237	0.04932	1081.24	0	Y	0
NUM1	-0.62868	0.25385	-2.48	0	X	3
NUM1,1	0.47258	0.62253	0.76	1	X	3
NUM1,2	0.73660	0.81006	0.91	2	X	3
DEN1,1	0.15411	0.90483	0.17	1	X	3
DEN1,2	0.27774	0.57345	0.48	2	X	3

Constant Estimate = 53.3223702

Variance Estimate = 0.70424109
Std Error Estimate = 0.83919073
AIC = 729.72486*
SBC = 751.764799*
Number of Residuals= 291
* Does not include log determinant.

Correlations of the Estimates

Variable	Parameter	Y MU	X NUM1	X NUM1,1	X NUM1,2	X DEN1,1	X DEN1,2
Y	MU	1.000	0.013	0.002	-0.002	0.004	-0.006
X	NUM1	0.013	1.000	0.755	-0.447	0.089	-0.065
X	NUM1,1	0.002	0.755	1.000	0.121	-0.538	0.565

X	NUM1,2	-0.002	-0.447	0.121	1.000	-0.892	0.870
X	DEN1,1	0.004	0.089	-0.538	-0.892	1.000	-0.998
X	DEN1,2	-0.006	-0.065	0.565	0.870	-0.998	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	496.45	6	0.000	0.893	0.711	0.502	0.312	0.167	0.064
12	498.58	12	0.000	-0.003	-0.040	-0.054	-0.040	-0.022	-0.021
18	539.38	18	0.000	-0.045	-0.083	-0.131	-0.170	-0.196	-0.195
24	561.87	24	0.000	-0.163	-0.102	-0.026	0.047	0.106	0.142
30	585.90	30	0.000	0.158	0.156	0.131	0.081	0.013	-0.037
36	592.42	36	0.000	-0.048	-0.018	0.038	0.070	0.079	0.067
42	593.44	42	0.000	0.042	0.025	0.013	0.004	0.006	0.019
48	601.94	48	0.000	0.043	0.068	0.084	0.082	0.061	0.023

Autocorrelation Plot of Residuals

```
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.704241    1.00000  |                    |********************|
  1   0.628846    0.89294  |                  . |******************  |
  2   0.500490    0.71068  |                .   |**************      |
  3   0.353404    0.50182  |                .   |**********          |
  4   0.219895    0.31224  |               .    |******              |
  5   0.117330    0.16660  |               .    |***  .              |
  6   0.044967    0.06385  |               .    |*    .              |
  7 -0.0023551   -0.00334  |               .    |     .              |
  8  -0.028030   -0.03980  |               .   *|     .              |
  9  -0.037891   -0.05380  |               .   *|     .              |
 10  -0.028378   -0.04030  |               .   *|     .              |
```

Partial Autocorrelations

```
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.89294  |                  . |******************  |
  2   -0.42765  |            *********| .                 |
  3   -0.13463  |                  ***| .                 |
  4    0.02199  |                  . | .                  |
  5    0.03891  |                  . |*.                  |
  6   -0.02219  |                  . | .                  |
```

```
  7   -0.02249  |                  . | .                    |
  8    0.01538  |                  . | .                    |
  9    0.00634  |                  . | .                    |
 10    0.07737  |                  . |**                    |
```

Crosscorrelation Check of Residuals with Input X

To Lag	Chi Square	DF	Prob	Crosscorrelations					
5	0.48	2	0.785	-0.009	-0.005	0.026	0.013	-0.017	-0.022
11	0.93	8	0.999	-0.006	0.008	0.022	0.023	-0.017	-0.013
17	2.63	14	1.000	0.012	0.035	0.037	0.039	-0.005	-0.040
23	19.19	20	0.509	-0.076	-0.108	-0.122	-0.122	-0.094	-0.041
29	20.12	26	0.786	-0.039	-0.013	0.010	-0.020	-0.031	-0.005
35	24.22	32	0.836	-0.022	-0.031	-0.074	-0.036	0.014	0.076
41	30.66	38	0.795	0.108	0.091	0.046	0.018	0.003	0.009
47	31.65	44	0.918	0.008	-0.011	-0.040	-0.030	-0.002	0.028

```
Model for variable Y

Estimated Intercept = 53.3223702

Input Number 1 is X with a shift of 3.

The Numerator Factors are
Factor 1: -0.6287 - 0.47258 B**(1) - 0.7366 B**(2)


The Denominator Factors are
Factor 1: 1 - 0.15411 B**(1) - 0.27774 B**(2)
```

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag	Variable	Shift
MU	53.26307	0.11926	446.63	0	Y	0
AR1,1	1.53292	0.04754	32.25	1	Y	0
AR1,2	-0.63297	0.05006	-12.64	2	Y	0

NUM1	-0.53522	0.07482	-7.15	0	X	3
NUM1,1	0.37602	0.10287	3.66	1	X	3
NUM1,2	0.51894	0.10783	4.81	2	X	3
DEN1,1	0.54842	0.03822	14.35	1	X	3

Constant Estimate = 5.32937102

Variance Estimate = 0.05882782
Std Error Estimate = 0.24254447
AIC = 8.29281119*
SBC = 34.0060741*
Number of Residuals= 291
* Does not include log determinant.

Correlations of the Estimates

Variable	Parameter	Y MU	Y AR1,1	Y AR1,2	X NUM1	X NUM1,1	X NUM1,2
Y	MU	1.000	-0.063	0.047	-0.008	-0.016	0.017
Y	AR1,1	-0.063	1.000	-0.927	-0.003	0.007	-0.002
Y	AR1,2	0.047	-0.927	1.000	0.023	-0.005	0.005
X	NUM1	-0.008	-0.003	0.023	1.000	0.713	-0.178
X	NUM1,1	-0.016	0.007	-0.005	0.713	1.000	-0.467
X	NUM1,2	0.017	-0.002	0.005	-0.178	-0.467	1.000
X	DEN1,1	-0.049	0.015	-0.022	-0.013	-0.039	-0.720

Variable	Parameter	X DEN1,1
Y	MU	-0.049
Y	AR1,1	0.015
Y	AR1,2	-0.022
X	NUM1	-0.013
X	NUM1,1	-0.039
X	NUM1,2	-0.720
X	DEN1,1	1.000

Autocorrelation Check of Residuals

```
 To    Chi                      Autocorrelations
Lag  Square DF   Prob
```

6	8.61	4	0.072	0.024	0.055	-0.073	-0.054	-0.054	0.119
12	15.43	10	0.117	0.032	0.028	-0.081	0.047	0.022	0.107
18	21.13	16	0.173	-0.038	0.052	-0.093	-0.013	-0.073	-0.005
24	27.52	22	0.192	-0.118	-0.002	-0.007	0.076	0.024	-0.004
30	36.94	28	0.120	0.034	-0.021	0.020	0.094	-0.118	0.065
36	44.26	34	0.112	-0.025	-0.057	0.113	0.022	0.030	0.065
42	45.62	40	0.250	-0.017	-0.036	-0.029	-0.013	-0.033	0.017
48	48.60	46	0.369	0.024	0.069	0.024	0.017	0.022	-0.044

Crosscorrelation Check of Residuals with Input X

To Lag	Chi Square	DF	Prob	Crosscorrelations					
5	0.93	3	0.819	0.008	0.004	0.010	0.008	-0.045	0.030
11	6.60	9	0.678	0.075	-0.024	-0.019	-0.026	-0.111	0.013
17	13.86	15	0.537	0.050	0.043	0.014	0.014	-0.141	-0.028
23	18.55	21	0.614	-0.074	-0.078	0.023	-0.016	0.021	0.060
29	27.99	27	0.411	-0.071	-0.001	0.038	-0.156	0.031	0.035
35	35.18	33	0.365	-0.014	0.015	-0.039	0.028	0.046	0.142
41	37.15	39	0.554	0.031	-0.029	-0.070	-0.006	0.012	-0.004
47	42.42	45	0.582	0.036	-0.038	-0.053	0.107	0.029	0.021

```
Model for variable Y

Estimated Intercept = 53.2630713

Autoregressive Factors
Factor 1: 1 - 1.5329 B**(1) + 0.63297 B**(2)


Input Number 1 is X with a shift of 3.

The Numerator Factors are
Factor 1: -0.5352 - 0.37602 B**(1) - 0.51894 B**(2)


The Denominator Factors are
Factor 1: 1 - 0.54842 B**(1)
```